Preventing the drop in security investments for non-competitive cyber-insurance market*

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Abstract. The rapid development of cyber insurance market brings forward the question about the effect of cyber insurance on cyber security. Some researchers believe that the effect should be positive as organisations will be forced to maintain a high level of security in order to pay lower premiums. On the other hand, other researchers conduct a theoretical analysis and demonstrate that availability of cyber insurance may result in lower investments in security.
In this paper we propose a mathematical analysis of a cyber-insurance model in a non-competitive market. We prove that with a right pricing strategy it is always possible to ensure that security investments are at least as high as without insurance. Our general theoretical analysis is confirmed by specific cases using CARA and CRRA utility functions.

1 Introduction

It is widely recognised that cyber security incidents are much more than just unpleasant events. Such incidents may cause huge losses (e.g., see effect of the latest two data breaches discoveries by Yahoo on its deal with Verizon[3]) and put in danger lives of people (e.g., cyber attacks on critical infrastructures). Therefore, the best risk management practices point out the need of considering cyber risk as a component of the overall risk management routine [23, 6, 21].

Unfortunately, installation of various countermeasures and adopting best cyber security practices do not guarantee freedom from cyber incidents, regardless their significant cost. In other words, organisations always face some residual cyber risks. The only option which was left for organisations so far was simply to accept this risk, i.e., acknowledge that such a problem may happen and, maybe, put some money aside to compensate the losses if the threat occurs (self-insurance). An alternative to these risk treatment options was introduced 20 years ago [12, 13]. This alternative is cyber insurance, a risk transfer option which allows insureds to shift their residual cyber risks to insurers.

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[3] http://www.euronews.com/2017/02/21/yahoo-pays-the-price-for-massive-data-breaches-in-verizon-deal.

Cyber insurance is believed to have a number of advantages, next to the obvious one, i.e., covering residual risks and smoothing possible losses. Cyber insurance is a means to collect statistics on cyber events and use it to evaluate security strength of various systems. The assigned premiums may serve as indicators of security strength [1]. Cyber insurance should increase the demand for cyber security standards [5]. Last but not least, cyber insurance is believed to be an intensive for organisations to invest in security in order to get lower premiums [17, 1, 12]. Unfortunately, some papers [15, 16, 20] show that without regulatory constraints competitive insurance is not an incentive for self-protection. In fact, the insureds prefer to insure their risks instead of mitigating them with investments. This puts other members of digital society under higher risks [8].

Several proposals were considered to find a solution for the problem and the best option found was "fine and rebate" regulation mechanism, which additionally fines insurers with low security and rebates the ones with high security, next to security discriminating strategy for assigning the premiums [11, 16]. These works consider an oversimplified model of security investment: an agent may invest in security a certain amount to get 100% protection of direct attack (but it still can be attacked indirectly, though contagion).

In this work, we propose another way of regulating cyber insurance market looking at the problem from the insured's point of view. We determine the minimal level of insurer's interests (loading factor) which guarantees that investments in insured's protection are as high as in case of no insurance available and insureds are still interested in transferring some (non-zero) part of their risks. Such enforcement may be introduced by the government as a tax for insurer, or by enforcing a law requiring the smallest insurer interest. We use a continuous model of security investments (similar to the one of Ogut et. al [15]) and consider a very generic class of utility functions for modelling insureds' satisfaction.

The result of our theoretical study is a system of two equations with two variables, i.e., it is solvable. On the other hand, because of generality of our approach the unique final formula is very hard to find (if possible at all). Nevertheless, if the utility function is known, it is possible to find the solution. We demonstrate this with our case studies using classical CARA and CRRA functions. In addition, we conduct several experiments to investigate the effect of security interdependence on insurance parameters.

The paper is structured as follows. Section 2 discusses the current achievements in the area and underlines the advantages of our approach. Section 3 introduces the basic insurance model, considering two cases: with cyber insurance available and without it. Section 4 contains our core contribution and describes how security investments can be raised with raise of premium. Section 5 shows two specific examples to confirm the theory. Section 6 outlines conclusions.

2 Related Work

Recently, cyber insurance has gained much attention in the scientific literature [2, 13]. Specific attention is devoted to the analysis of the effect of interdependent security on cyber insurance [15, 10, 19, 16, 9]. In fact, strong influence of security interdependency is one of the main features that make cyber insurance a specific insurance case.

Ogut et al., [15] provided an analysis in depth of the interdependent security and immaturity of the market on cyber insurance. In particular, the authors investigated how investments in self-protection change. They have found that these investments in self protection reduce with growth of the interdependence and these investments rise with growth of immaturity of insurance market. Moreover, the authors considered the effect of enforcement of liability for contagion. They have found, that investments in self-protection in this case rise even higher than the optimal level. The results of the study are limited because of the following assumptions: 1) the authors use only CARA as a utility function; 2) the losses are considered to be too small with respect to the wealth of insureds. The last assumption is particularly dangerous for insurance, since it significantly reduces the effect of risk averseness of insureds. In contrast, we provide a generic approach without the outlined assumptions (we use CARA as the utility function only as an example). Moreover, we show how it is possible to compute the loading factor value to achieve the desired level of self-investment.

A number of authors considered the problem of reducing self-protection level if cyber insurance is available [10, 19, 16] and whether the optimum level of investments can be reached [20, 18]. The solution for the problem found by several authors is additional fines/rebates for the users with low/high security [11, 3, 10, 16]. Naturally, in this case the insurer has to know exactly the level of investments by insureds, i.e., no information asymmetry is allowed (similar to our assumption). Here we should point out that in these cases the authors consider a discrete model of investments, which has two levels (with low protection and fully protected against direct attacks) and specific level of investments required to jump from one level to another one. In reality investments in cyber security have more levels or have continuous impact on probability of an incident.

An interesting method was proposed by P. Naghizadeh and M. Liu [14] for specifying the optimal level of investments. In the proposed model the insurer collects the proposals of all its insureds (the whole society) about the desired level of investments and adjusts the policies (i.e., premiums) correspondingly. The authors show that they are able to reach the optimal level with this approach, if participation of all agents in such schema is mandatory. In contrast, we consider voluntary participation and ensure that with specified price of insurance the agents are still interested in buying the policy ($I \geq 0$).

3 Basic formalisation

Before we go into the discussion of our basic problem, we specify the basic formalisation. We define only the concepts required for our paper and refer the reader interested in the comprehensive definition of basic terms to [13].

Let W^0 be the amount of wealth an agent possesses now. The agent tries to predict its wealth after some period of time (typically, in a year). Naturally, the agent does not know if a threat causing losses to him/her will occur during the considered period, but it may invest some amount of money x to decrease the probability of the incident. Because of the uncertainty about the final outcome, the value is random (and is denoted as $\boldsymbol{W}$), but it is possible to make some predictions about it if the probability of $pr(x)$ and the loss L caused by the incident are known. We see that $pr(x)$ depends on x, i.e., the probability of the incident depends on the amount of investments[4]. It is natural to assume that higher investments lead to lower probability of occurrence: $pr'(x) < 0$; but lower initial investment level requires less additional investments to decrease the probability of occurrence by the same value: $pr''(x) < 0$. The final loss in this case is also a random variable $\boldsymbol{L}$ and is equal either to L, if the threat occurs, or to 0, otherwise.

The expected wealth $E[\boldsymbol{W}]$ after the considered period could be computed as[5]:

$$E[\boldsymbol{W}] = W^0 - E[\boldsymbol{L}] - x = W^0 - pr'(x)(L) + (1 - pr'(x))(0) - x. \quad (1)$$

Let $U(W)$ be a function of wealth, and can be seen as the satisfaction of agents to posses a certain amount of money. The utility function is not linear, and in many situations, increase in satisfaction is lower for higher amount of wealth possessed [22]. Such behaviour of an agent is called risk averseness and can be modelled with a utility function satisfying the following conditions: $U'(W) > 0$ and $U''(W) < 0$. Instead of expected wealth (Equation 1), we should look now for the expected utility of wealth:

$$E[U(\boldsymbol{W})] = pr(x) * U_L(W^0 - L - x) + (1 - pr(x))U_N(W^0 - x). \quad (2)$$

In this paper, we use a similar formalisation to Ogut et. al [15], which is very generic. Nevertheless, in our work losses could be very high, and the utility function is not bound to be a constant absolute risk aversion (CARA) function only.

3.1 No-insurance case

First, we consider the situation when insurance is not available for agents. Let,

$$U_{NN} = U(W^0 - x) \text{ if no incident occurs;} \quad (3)$$

$$U_{NL} = U(W^0 - L - x) \text{ if an incident occurs.} \quad (4)$$

[4] We acknowledge that in reality effect of investments on probability of occurrence is more complex and an incident may occur more than once but we would like to underline that this standard (for cyber investment models [8, 7, 10, 16, 13] and general insurance [4]) modelling is an approximation of reality which reduces the complexity of computations and allows to analyse the core insights[7].

[5] Although, the Equation 1 can be simplified, we leave it in this form to underline the similarity with the following step in the discussion.

The expected utility in this case is:

$$E[U(\boldsymbol{W})] = pr(x) * U_{NL} + (1 - pr(x))U_{NN}. \tag{5}$$

We take the first order condition (FOC) for x and look for optimal solution x^N.

$$\frac{\partial E[U(\boldsymbol{W})]}{\partial x} =$$

$$pr'(x^N)U_{NL} - pr(x^N)U'_{NL} - (1 - pr(x^N))U'_{NN} - pr'(x^N)U_{NN} = 0; \tag{6}$$

$$pr'(x^N)(U_{NL} - U_{NN}) = pr(x^N) * U'_{NL} + (1 - pr(x^N))U'_{NN}; \tag{7}$$

$$pr'(x^N) = \frac{pr(x^N) * U'_{NL} + (1 - pr(x^N))U'_{NN}}{(U_{NL} - U_{NN})}. \tag{8}$$

The solution to Equation 8 will provide us with the optimal amount of money an agent should invest in self-protection if insurance is not available.

3.2 Competitive insurance market

Now, we consider the situation when insurance is available to agents. An insurer agrees to bare some part of insured's loss, called an indemnity I ($I \leq L$), in case an incident occurs. An insured pays the premium P as a fee for this service. The premium is usually linked to indemnity by the following relation:

$$P = (1 + \lambda) * pr(x)I; \tag{9}$$

where λ is the degree of market immaturity. This degree can be seen as the amount of money the insurer may ask for the service it provides. If insurance market is mature, i.e., it is a competitive market, it is assumed that no insurers are able to provide a better insurance product than others already do, and $\lambda = 0$.

First we introduce the utility functions for insurance case.

$$U_{IN} = U(W^0 - pr(x)(1 + \lambda)I - x) \quad \text{if no incident occurs;} \tag{10}$$

$$U_{IL} = U(W^0 - L + I - pr(x)(1 + \lambda)I - x) \quad \text{if an incident occurs.} \tag{11}$$

The expected utility in this case is

$$E[U(\boldsymbol{W})] = pr(x) * U_{IL} + (1 - pr(x))U_{IN}. \tag{12}$$

In case of a competitive insurance market and using Equation 12, it is possible to find the optimal level of investment, which is equal to[6]

$$x^I = -\frac{1}{L}. \tag{13}$$

[6] See the proof in [15] or [4].

Comparing insurance x^I and no-insurance cases x^N we would like to be sure that security investments will increase the security level of insured. Formally,

$$pr'(x^N) \leq pr'(x^I) \quad or \quad pr'(x^N) \leq -\frac{1}{L}. \tag{14}$$

We see that if this condition holds, security investments, even in case of a competitive market, are higher than in case of no-insurance. On the contrary, many studies [15, 16, 20] show that cyber insurance tends to reduce the investments and

$$pr'(x^N) > -\frac{1}{L}. \tag{15}$$

In other words, the presence of (competitive) insurance worsens the security investment level for an agent.

Our goal is to investigate the possibility to raise the security investments up to the level of no-insurance, by raising loading factor. The later can be achieved by special taxes applied to insurer to ensure that the loading factor λ is high enough to incentivise insureds to invest in cyber security. Moreover, we will ensure that agents would like to buy insurance regardless the increased price, i.e., the coverage (indemnity) $I > 0$.

4 Raising security investment level with insurance

Since an insured would like to maximise its utility her/she will set up the best I and x. In other words, we should consider the first order conditions of Equation 12 for I and x and find the optimal values $I^\star$ and $x^\star$.

$$\frac{\partial E[U(\boldsymbol{W})]}{\partial I} = \\ pr(x) * U'_{IL}(1 - pr(x)(1+\lambda)) - (1 - pr(x))(1+\lambda)pr(x)U'_{IN} = 0. \tag{16}$$

From Equation 16 it follows that:

$$\frac{U'_{IL}}{U'_{IN}} = \frac{(1 - pr(x))(1+\lambda)}{1 - pr(x)(1+\lambda)} \ or \tag{17}$$

$$1 + \lambda = \frac{U'_{IL}}{U'_{IN}(1 - pr(x)) + pr(x)U'_{IL}}. \tag{18}$$

We can do the similar analysis for investments.

$$\begin{aligned}\frac{\partial E[U(\boldsymbol{W})]}{\partial x} =& pr'(x^\star) * U_{IL} - pr(x^\star) * U'_{IL}(pr'(x^\star)(1+\lambda)I + 1) - \\ & (1 - pr(x^\star))U'_{IN}(pr'(x^\star)(1+\lambda)I + 1) - pr'(x^\star)U_{IN} = 0.\end{aligned} \tag{19}$$

With some simple transformations, we come to the following form:

$$\frac{(U_{IL} - U_{IN})}{(pr(x^\star)U'_{IL} + (1 - pr(x^\star))U'_{IN})} - \frac{1}{pr'(x^\star)} = (1 + \lambda)I. \tag{20}$$

Our goal is to achieve the same level of security investments as in case of no insurance available, i.e., $x^\star = x^N$. Moreover, in this case the amount of insurance bought must be optimal $I = I^\star$. Naturally, the solution to our problem $(\lambda, I^\star)$ is the solution to the following system of equations:

$$\begin{cases} 1 + \lambda = \frac{U'_{IL}}{U'_{IN}(1-pr(x^\star))+pr(x^\star)U'_{IL}}; \\ \frac{(U_{IL}-U_{IN})}{(pr(x^\star)U'_{IL}+(1-pr(x^\star))U'_{IN})} - \frac{1}{pr'(x^\star)} = (1+\lambda)I^\star. \end{cases} \tag{21}$$

Although this is a system of two equations with two variables, its solution is not easy to find in the current form. As we will show in the following (see Section 5), the solution is not simple (but is possible) even when all functions and values are precisely defined. The main question we would like to answer is whether this system has a non-zero solution for indemnity, i.e, $I^\star > 0$ if $(\lambda, I^\star)$ is the solution for Equation 21.

Theorem 1. *If the level of investments in self-protection for the competitive cyber insurance market is lower than in case of no insurance available and if the utility function of insured is a of decreasing absolute risk aversion (DARA) type, there is such a setting of λ for non-competitive cyber insurance market which ensures that*

1. *the level of investments is equal to the case of no insurance ($x^\star = x^N$);*
2. *the amount of insurance bought is higher than zero ($I^\star > 0$).*

Proof. First, lets put Equation 18 to Equation 20:

$$\frac{(U_{IL} - U_{IN})}{(pr(x^\star)U'_{IL} + (1 - pr(x^\star))U'_{IN})} - \frac{1}{pr'(x^\star)} = \frac{U'_{IL}}{U'_{IN}(1 - pr(x)) + pr(x)}I; \tag{22}$$

$$-\frac{1}{pr'(x^\star)} + \frac{(U_{IL} - U_{IN} - I^\star U'_{IL})}{(pr(x^\star)U'_{IL} + (1 - pr(x^\star))U'_{IN})} = f(I^\star) = 0. \tag{23}$$

Now we investigate the function $f(I^\star)$. If we consider $x^\star = x^N$, it is easy to see that $I^\star = 0$ is a solution to the Equation 23. Trivially, if an agent decides not to buy insurance ($I^\star = 0$) then its optimal investment level is the same as in case when no insurance is available. What we are interested in is *whether there is another solution for Equation 23 on the interval $I^\star \in [0, L]$*.

Another important observation we can make is for the other extreme, when $I^\star = L$, i.e., full insurance case. We see from Equations 10 and 11 that in this case: $U_{IL} = U_{IN}$ and the right summand of $f(I^\star)$ is equal to $-L$. Recalling the assumption from Equation 15, we see that:

$$\frac{1}{pr'(x^\star)} = \frac{1}{pr'(x^N)} < -L. \tag{24}$$

From Equation 24 it follows that $f(I^\star)|_{I^\star=L} > 0$. Although $I = L$ is not a solution, we get some information about the behaviour of $f(I^\star)$ function.

We have found that $f'(I^\star)|_{I^\star=0} < 0$[7]. Since, we know, that $f(I^\star)|_{I^\star=L} > 0$ and the function is continuous (on the interval $I^\star \in [0; L]$[8]), then according to the Intermediate Value Theorem there *must be at least one more point with $I^\star > 0$ which is the solution to Equation 23 (the point, where function $f(I^\star) = 0$* for $I^\star \in (0; L)$).

Insureds prefer to buy insurance. We have shown above that there are at least two solutions to our problem: with $I^* = 0$ and $I^* > 0$.

Insurers, clearly, would like to have $I^* > 0$, and thus, set λ to ensure this choice of the insured. Consider this problem also from the insured point of view. The insured will always select the strategy which maximises its utility $E[U(\boldsymbol{W})]$. Moreover, since the strategy "do not buy insurance" is always available in our settings, we would like to be sure that the solution for $I^* > 0$ is preferable. Compare these two cases:

$$\begin{aligned}
&E[U(W)]|_{I^\star\neq 0} - E[U(W)]|_{I^\star=0} = \\
&pr(x^N)U_{IL} + (1 - pr(x^N))U_{IN} - pr(x^N)U_{NL} - (1 - pr(x^N))U_{NN} = \\
&pr(x^N)(U_{IL} - U_{NL}) + (1 - pr(x^N))(U_{NN} - U_{IN}).
\end{aligned} \tag{25}$$

Now, we recall that $U_{IL} \geq U_{NL}$ and $U_{NN} \geq U_{IN}$, while the utility function is convex, i.e., $U_{IL} - U_{NL} < U'_{IL}(I^\star(1 - pr(x^N)(1 + \lambda)))$ and $U_{NN} - U_{IN} > U'_{IN}(I^\star pr(x^N)(1 + \lambda))$. Finally, using Equation 17 we find that the result is greater than 0.

$$\begin{aligned}
&E[U(W)]|_{I^\star\neq 0} - E[U(W)]|_{I^\star=0} \geq \\
&pr(x^N)U'_{IL}(I^\star(1 - pr(x^N)(1 + \lambda))) - (1 - pr(x^N))U'_{IN}(I^\star pr(x^N)(1 + \lambda)) = 0.
\end{aligned} \tag{26}$$

We conclude that, for any $I^\star$, $E[U(W)]|_{I^\star\neq 0} \geq E[U(W)]|_{I^\star=0}$, i.e., an insured always prefers to buy some insurance if the settings are as specified by solution of Equation 21.

It is easy to see that if $\lambda = 0$ than $I^\star = L$. Now, if $I^\star = 0$, then

$$\lambda = \frac{(U'_{NL} - U'_{NN})(1 - pr(x))}{(U'_{NL} - U'_{NN})pr(x) + U'_{NN}}. \tag{27}$$

Out of Equation 27, we conclude that the loading factor to force the security level to be equal to x^N belongs to the interval $[0; \frac{(U'_{NL}-U'_{NN})(1-pr(x))}{(U'_{NL}-U'_{NN})pr(x)+U'_{NN}}]$.

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[7] See the proof in the Appendix

[8] $f'(I^\star)$ is continuous on the interval $I^\star \in [0; L]$ since neither $pr'(x^\star) = 0$ nor $(pr(x^\star)U'_{IL} + (1 - pr(x^\star))U'_{IN}) = 0$ for realistic values.

Using Equation 17 for $I \neq 0$ and for $I = 0$ it is easy to find that the loading factor in the first case is always lower:

$$\frac{(1-pr(x))}{pr(x)+\frac{1}{\frac{U'_{NL}}{U'_{NN}}-1}} \geq \frac{(1-pr(x))}{pr(x)+\frac{1}{\frac{U'_{IL}}{U'_{IN}}-1}}, \quad since\ U'_{IL} \geq U'_{NL}\ and\ U'_{IL} \leq U'_{NL}. \tag{28}$$

4.1 Interdependence of security.

Until now we considered only an independent case, i.e., security level of one agent did not depend on the security level of another one. In the cyber world this is not usually the case. Thus, we should change our model of probability to:

$$pr_i(x_i, X_{-i}) = 1 - (1 - \pi_i(x_i)) * \mathit{\Pi}_{-i}. \tag{29}$$

where $\mathit{\Pi}$ is the degree of the network security. In other words, $\mathit{\Pi}$ determines the probability that the agent will be compromised indirectly, i.e., through other member of the network. In cyber insurance literature it is usually equals to:

$$\mathit{\Pi}_{-i} = \prod_{\forall j \neq i} (1 - q * \pi_j(x_j)). \tag{30}$$

In this paper, we focus on the effect of the overall network security on a concrete insured. Therefore, in our study insurance (as a risk treatment option) is available only for this insured. Thus, we omit indexes i and $-i$ and skip X_{-i}.

It is easy to see that the procedure for finding I and λ does not change much, we simply should use Equation 30 instead of simple $pr(i)$.

$$\begin{cases} 1+\lambda = \frac{U'_{IL}}{U'_{IN}((1-\pi(x^\star))*\mathit{\Pi})+(\pi(x^\star)*\mathit{\Pi})U'_{IL}}; \\ \frac{(U_{IL}-U_{IN})}{U'_{IN}((1-\pi(x^\star))*\mathit{\Pi})+(\pi(x^\star)*\mathit{\Pi})U'_{IL}} - \frac{1}{\pi'(x^\star)*\mathit{\Pi}} = (1+\lambda)I^\star. \end{cases} \tag{31}$$

5 Examples and analysis of CARA and CRRA

Since the found solution is quite complex in its generic view, in this section we will demonstrate how the finding can be applied in specific cases of the two DARA utility functions most frequently applied for cyber insurance [13]: Constant Absolute Risk Aversion (CARA) and Constant Relevant Risk Aversion (CRRA) functions. We would like to underline that CARA and CRRA utility functions are only useful examples, while the findings from Section 4 are valid for any concave utility function.

5.1 CARA utility function

Constant Absolute Risk Aversion (CARA) utility function is a function for which the following relation holds:

$$-\frac{U''(W)}{U'(W)} = \sigma; \quad \sigma > 0. \tag{32}$$

The unique function satisfying this relation is the exponential function:

$$U(W) = 1 - \exp^{-\sigma W}; \quad U'(W) = \sigma \exp^{-\sigma W}; \quad U''(W) = -\sigma^2 \exp^{-\sigma W}. \quad (33)$$

If we apply this utility function to our Equation system 21, then using the first equation, we can find that:

$$e^{\sigma(L-I^\star)} = \frac{(1+\lambda)(1-pr(x^\star))}{(1-pr(x^\star)(1+\lambda))} \quad or \quad (34)$$

$$I^\star = L - \frac{1}{\sigma} ln\left[\frac{(1+\lambda)(1-pr(x^\star))}{(1-pr(x^\star)(1+\lambda))}\right]. \quad (35)$$

The second equation from the system can be changed to:

$$\frac{1}{\sigma}\frac{1-e^{\sigma(L-I^\star)}}{1-pr(x^\star)+pr(x^\star)(e^{\sigma(L-I^\star)})} - \frac{1}{pr'(x^\star)} = (1+\lambda)I^\star \quad or \quad (36)$$

$$\frac{1}{\sigma}\frac{\lambda}{1-pr(x^\star)} - \frac{1}{pr'(x^\star)} = (1+\lambda)I^\star. \quad (37)$$

$f(I^\star)$ function from Equation 23 assumes the following form:

$$\frac{1}{\sigma}\frac{\lambda}{1-pr(x^\star)} - \frac{1}{pr'(x^\star)} - (1+\lambda)I^\star = f(I^\star). \quad (38)$$

Now, it is possible to see that the loading factor (λ) we are looking for is the solution of the following equation.

$$\frac{1}{\sigma(1+\lambda)}\frac{\lambda}{1-pr(x^\star)} - \frac{1}{pr'(x^\star)(1+\lambda)} = L - \frac{1}{\sigma} ln\left[\frac{(1+\lambda)(1-pr(x^\star))}{(1-pr(x^\star)(1+\lambda))}\right]. \quad (39)$$

Equation 39 is still hard to solve theoretically. Viable approaches are graphic solutions or approximation algorithms.

5.2 CRRA utility function

Constant Relative Risk Aversion (CRRA) utility function is a function for which the following relation holds:

$$-\frac{U''(W)}{U'(W)} = \frac{\sigma}{W}; \quad \sigma > 0. \quad (40)$$

The utility function itself can de defined as follows:

$$U(W) = \begin{cases} \frac{W^{1-\sigma}}{1-\sigma} & \text{for } \sigma \neq 1 \\ log(W) & \text{for } \sigma = 1 \end{cases}; \quad U'(W) = W^{-\sigma}; \quad U''(W) = -\sigma\frac{W^{-\sigma}}{W}. \quad (41)$$

Without loss of generality, we assume that $\sigma \neq 1$.

If we apply this utility function to our Equation system 21, then using the first equation, we can find that:

$$\left(\frac{W^0 - pr(x^\star)(1+\lambda)I^\star - x^\star}{W^0 - L + I - pr(x^\star)(1+\lambda)I^\star - x^\star}\right)^\sigma = \frac{(1+\lambda)(1-pr(x^\star))}{(1-pr(x^\star)(1+\lambda))} = \alpha \quad or \quad (42)$$

$$I^\star = \frac{L\sqrt[\sigma]{\alpha} - (W^0 - x^\star)(\sqrt[\sigma]{\alpha} - 1)}{\sqrt[\sigma]{\alpha} - pr(x^\star)(1+\lambda)(\sqrt[\sigma]{\alpha} - 1)}. \quad (43)$$

The second equation from the system can be changed to:

$$\frac{1}{1-\sigma}\frac{(W^0 - pr(x^\star)(1+\lambda)I^\star - x^\star)(\alpha - 1) - \alpha L + \alpha I^\star}{pr(x^\star)\alpha + (1 - pr(x^\star))} - \frac{1}{pr'(x^\star)} = (1+\lambda)I^\star;$$

$$I^\star = \frac{pr'(x^\star)((W^0 - x^\star)(\alpha - 1) - \alpha L) - \beta}{pr'(x^\star)((1+\lambda)\beta + pr(x^\star)(1+\lambda)(\alpha - 1) - \alpha)}, \quad (44)$$

$$where\ \beta = (1-\sigma)(pr(x^\star)\alpha + (1 - pr(x^\star))).$$

$f(I^\star)$ function from Equation 23 assumes the following form:

$$f(I^\star) = \frac{(W^0 - pr(x^\star)(1+\lambda)I^\star - x^\star)(\alpha - 1) - \alpha L + \alpha I^\star}{(1-\sigma)(pr(x^\star)\alpha + (1 - pr(x^\star)))} - \frac{1}{pr'(x^\star)} - (1+\lambda)I^\star. \quad (45)$$

Now, it is possible to see that the loading factor (λ) we are looking for is the solution of the following equation.

$$\frac{L - (W^0 - x^\star)(\sqrt[\sigma]{\alpha} - 1)}{\sqrt[\sigma]{\alpha} - pr(x^\star)(1+\lambda)(\sqrt[\sigma]{\alpha} - 1)} = \frac{pr'(x^\star)((W^0 - x^\star)(\alpha - 1) - \alpha L) - \beta}{pr'(x^\star)(1+\lambda)\beta + pr(x^\star)(1+\lambda)(\alpha - 1) - \alpha}. \quad (46)$$

Equation 46 is still hard to solve theoretically. Viable approaches are graphic solutions or approximation algorithms.

5.3 Numerical analysis

Finally, we would like to demonstrate the correctness of our theoretical approach with a couple of numerical examples. The initial wealth is assumed to be 20 (thousand) euro, and a possible loss estimated to be around 10 (thousand) euro. In both considered cases (for CARA and CRRA utility functions) we use the same $\sigma = 0.1$. We define the probability function as follows (ensuring that $pr'(x) < 0$ and $pr''(x) < 0$):

$$pr(x) = \frac{0.2}{(1+x)}. \quad (47)$$

With these settings, we are now able to find the resulting λ and I.

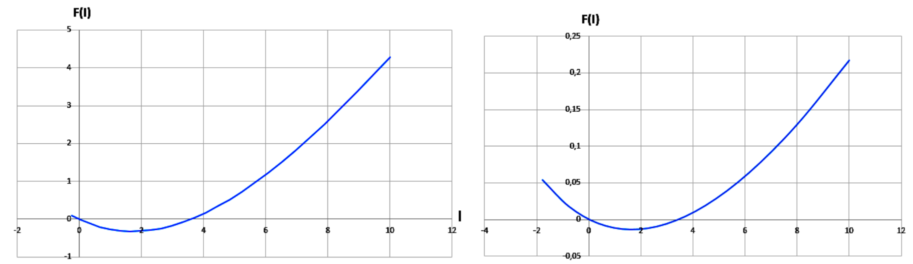

Fig. 1. $f(I)$ for CARA and CRRA examples.

CARA We start with the example of CARA utility function with $\sigma = 0.1$. First of all we solve Equation 8 to obtain x^N, which is also our target level of security investments in the insurance case $x^N = x^\star$. Now, when the probability function is known (Equation 47), Equation 8 can be transformed to a quadratic equation with one solution always negative. The second solution in our case is $x^N \approx 0.69$. If we compute $pr'(x^N)$ we will see that it satisfies condition stated in Equation 15 $pr'(x^N) \approx -0.07 > -1/10 = -0.1$.

First, consider the auxiliary function $f(I^\star)$ and its behaviour (see Equation 38). The left part of Figure 1 shows the behaviour of the function. As we found out in our reasoning in Section 4, the function crosses the line $f(I^\star) = 0$, when $I^\star = 0$ and $\lambda \approx 1.26$. Moreover, there is also at least one more intersection with this line for $I^\star \approx 3,5946 \neq 0$ and $\lambda \approx 0.7153$.

Naturally, there is no need to consider this auxiliary function looking for the optimal values. It is more convenient to consider Equation 39 and find the intersection points of left and right parts of the equation. The left part of Figure 2 shows these functions. In order to find the resulting values of $I^\star$ and λ we applied a simply hybrid root-finding algorithm[9].

CRRA We conducted a similar analysis for CRRA utility function with the same $\sigma = 0.1$. The found level of investment is $x^N \approx 0.43$, which also satisfies condition in Equation 15 $pr'(x^N) \approx -0,0978 > -1/10 = -0.1$.

Then, we found $f(I^\star)$ using Equation 45 (the right part of Figure 1) and intersection of left and right hand parts of Equation 46 (the right part of Figure 2). This function also crosses the line $f(I^\star) = 0$, when $I^\star = 0$ and $\lambda \approx 0.063$, plus, there is an intersection for $I^\star \approx 3.4586 \neq 0$ when $\lambda \approx 0.0267$.

Effect of Interdependency. Consider now the effect of interdependency (using the degree of network security Π from Equation 30) on the incentive to buy cyber

[9] First, we cut the considered interval into small pieces and found the pieces with border values of different signs. Then, we applied bisection method, cutting the piece in half and checking the signs of the function on border values, always leaving the half with different signs of the function on the border until the last half is shorter than the allowed error.

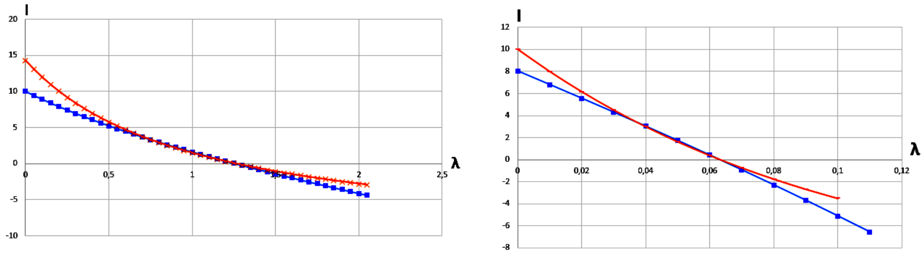

Fig. 2. Intersections of $I(\lambda)$ for Equations 39(left) and 46 (right).

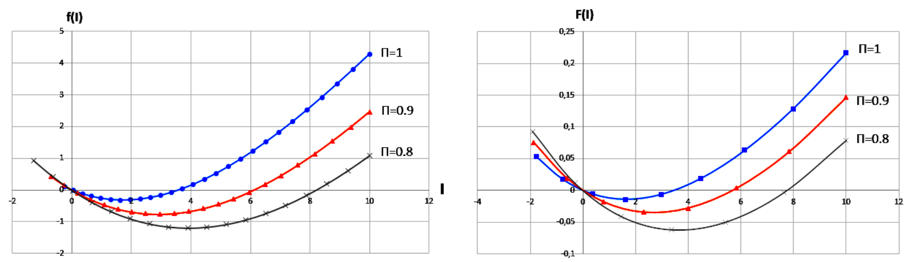

Fig. 3. $f(I)$ for CARA (left) and CRRA (right) examples with different degree of interdependency.

insurance. We conducted an analysis with three coefficients: $\Pi = 1$, $\Pi = 0.9$, and $\Pi = 0.8$. Figure 3 shows the result.

It is important to note that with the fall of security of the overall network and the growth of the interdependence (causing the fall of Π) the agents are more willing to buy insurance. This can be seen in the graphs, as the point where $f(I)$ line crosses axis I (i.e., $f(I) = 0$) shifts left, and the agent prefers to buy more insurance. Moreover, there is the required increase in the insurance cost (i.e., λ) is lower: for CARA $\lambda = 0.1131$ and for CRRA $\lambda = 0.00947$ if $\Pi = 0.8$ vs. for CARA $\lambda = 0.7153$ and for CRRA $\lambda = 0.0267$ if $\Pi = 1$.

Finally, we see that our function gets lower and lower with increase of interdependency effect, and, eventually, its right end gets below 0. This indicates that the optimal investments in case of buying cyber insurance with fair price become higher than in optimal investments in case cyber insurance is not available.

6 Conclusion

In the paper we have studied the possibility to ensure that investment level with available cyber insurance is at least as high as if cyber insurance was not available. This was achieved by the means of increasing the costs of cyber insurance. Our generic analysis have shown that the equal level with no-insurance case is always possible and regardless the higher prices insureds are still interested in buying some portion of insurance. Here we would like to underline, that in contrast to other researchers [15] our analysis has much less assumptions for the modelling. The high enough price for insurance can be enforced by the regulatory body either as a minimal price or with some special tax.

We conducted some numerical experiments with two case studies, where CARA and CRRA functions have been used. The experiments support our findings. Moreover, we were able to find that agents are more eager to buy more insurance with increase of interdependency effect. Furthermore, although the investments in self-protection fall with increase of interdependency effect, agents also become more incentivised by cyber insurance in comparison with no-insurance case. The latest observation requires more thorough theoretical research to prove the dependency for all cases, but we leave it for the future work.

As a future work, we see great potential in the considered model to study the possibility to affect investment level with adjusting the price. For example, we are able to raise investment level even higher than in no-insurance case. Thus, it would be nice to find the maximal value of investments which can be reached. Moreover, currently we considered the network as something that is not affected by available cyber-insurance. In the future, we would like to consider how the security investments change if other participants of the network also have the possibility to insure themselves. Last but not least, we would like to investigate how the analysed mechanism behaves in the models with information asymmetry, i.e., where moral hazard and adverse selection problems have place.

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7 Appendix

We prove that $f'(I^\star)|_{I^\star=0} < 0$.

Proof.

$$\frac{df}{dI^\star} =$$

$$\frac{\left[(1-(1+\lambda)pr(x^\star) - pr(x^\star)I\frac{d\lambda}{dI^\star})U'_{IL} - U'_{IL}\right]\left[pr(x^\star)U'_{IL} + (1-pr(x^\star))U'_{IN}\right]}{(pr(x^\star))U'_{IL} + (1-pr(x^\star))U'_{IN})^2} +$$

$$\frac{\left[(1+\lambda)pr(x^\star)+pr(x^\star)I\frac{d\lambda}{dI^\star}\right]U'_{IN}\left[pr(x^\star)U'_{IL}+(1-pr(x^\star))U'_{IN}\right]}{(pr(x^\star))U'_{IL}+(1-pr(x^\star))U'_{IN})^2}-$$
$$\frac{I^\star\left[1-(1+\lambda)pr(x^\star)-pr(x^\star)I\frac{d\lambda}{dI^\star}\right]U''_{IL}\left[pr(x^\star)U'_{IL}+(1-pr(x^\star))U'_{IN}\right]}{(pr(x^\star))U'_{IL}+(1-pr(x^\star))U'_{IN})^2}-$$
$$\frac{\left[U_{IL}-U_{IN}-I^\star U'_{IL}\right]pr(x^\star)\left[1-(1+\lambda)pr(x^\star)-pr(x^\star)I\frac{d\lambda}{dI^\star}\right]U''_{IL}}{(pr(x^\star))U'_{IL}+(1-pr(x^\star))U'_{IN})^2}-$$
$$\frac{\left[U_{IL}-U_{IN}-I^\star U'_{IL}\right]pr(x^\star)(1-pr(x^\star))\left[-(1+\lambda)pr(x^\star)-pr(x^\star)I\frac{d\lambda}{dI^\star}\right]U''_{IN}}{(pr(x^\star))U'_{IL}+(1-pr(x^\star))U'_{IN})^2}. \tag{48}$$

What we are interested in is the sign of the first derivative when $I^\star = 0$. Since the divisor is clearly grater than zero, we focus on the dividend only. $U_{IL}|_{I^\star=0} = U_{NL}$ and $U_{IN}|_{I^\star=0} = U_{NN}$ and derivatives. We reduce the first part of Equation 48 by U'_{IL} inside the first brackets. The third part is 0, as well as all subparts with $\frac{d\lambda}{dI^\star}$. In the last part we move out $pr(x^\star)(1-(1+\lambda)pr(x^\star))$. We get:

$$\begin{aligned}
&(1+\lambda)pr(x^\star)(-U'_{NL}+U'_{NN})(pr(x^\star)U'_{NL}+(1-pr(x^\star))U'_{NN})+\\
&(U_{NN}-U_{NL})pr(x^\star)(1-(1+\lambda)pr(x^\star))[(U''_{NL}-\frac{(1-pr(x^\star))(1+\lambda)}{(1-(1+\lambda)pr(x^\star))}U''_{NN})]=\\
&(1+\lambda)pr(x^\star)(-U'_{NL}+U'_{NN})(pr(x^\star)U'_{NL}+(1-pr(x^\star))U'_{NN})+\\
&(U_{NN}-U_{NL})pr(x^\star)(1-(1+\lambda)pr(x^\star))[(U''_{NL}U'_{NN}-U''_{NN}U'_{NL})]\frac{1}{U'_{NN}}.
\end{aligned} \tag{49}$$

We know, that $U'_{NL} > U'_{NL}$ and the first derivative is positive. Thus, the first summand is negative. Also $U'_{NL} < U'_{NL}$ and utility function is always positive. Also, $1 > (1+\lambda)pr(x^\star)$, otherwise an insured should pay more premium than the identity it gets in case of an incident. The only part left for consideration is $(U''_{NL}U'_{NN} - U''_{NN}U'_{NL})$.

We would like to recall that for the utility functions in use a *coefficient of absolute risk aversion* is defined as:

$$A(\boldsymbol{W}) = -\frac{U''(\boldsymbol{W})}{U'(\boldsymbol{W})}. \tag{50}$$

Moreover, the experimental and empirical evidence mostly confirm the decreasing absolute risk aversion (DARA). For the sake of generality, here we assume non-increasing risk aversion (CARA and DARA):

$$\frac{\partial A(\boldsymbol{W})}{\partial \boldsymbol{W}} \le 0. \tag{51}$$

In other words $A(W_{NL}) \ge A(W_{NN})$, where W_{NL} is the financial position of an insured in case of incident, while W_{NL} is the financial position of an insured in case no incident happens.

Thus, $(U''_{NL}U'_{NN} - U''_{NN}U'_{NL}) = U'_{NN}U'_{NL}[A(W_{NN}) - A(W_{NL})] \le 0$ and the second summand in the overall formula is negative or zero.