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Optimal energy vs. delay trade off in opportunistic networks

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Abstract—Opportunistic communications have been recently proposed as a key strategy for offloading traffic from 3G/4G cellular networks, which is particularly beneficial in case of crowded areas where many users are interested in similar contents. To conserve energy, duty cycling schemes are typically applied, and therefore contacts between nodes may become intermittent and sporadic also in dense networks. It is thus of paramount importance to accurately tune the duty cycling policy in order to meet energy requirements without compromising the quality of communications. In this paper, building upon a model of duty cycling in opportunistic networks that we have validated in a previous work, we study how to optimise the value of the duty cycle in order to provide probabilistic guarantees on the delay experienced by messages. More specifically, for a broad range of end-to-end delay distributions, and for the most relevant cases of inter-contact times distributions found in real traces, we provide closed-form approximated solutions for deriving the optimal duty cycle such that the probability that the delay is smaller than a target value z is greater than or equal to a configurable probability p.

Index Terms—Computer Society, IEEEtran, journal, LATEX, paper, template.

1 Introduction

Opportunistic networks have been conceived at the intersection between Mobile Ad hoc NETworks (MANET) and the Delay Tolerant (DTN) paradigm. In the conventional model, they exploit the movements of the nodes of the network (people with their smart, handheld devices like tablets and smartphones) in order to deliver messages to their destinations according to the store-carry-and-forward paradigm: nodes hold messages while they move and forward them to other nodes that are in radio contact, until messages reach their final destination. Opportunistic communications were initially seen as a standalone solution for those scenarios in which the nodes of the network were sparse and the infrastructure unavailable (disaster/emergency scenarios, developing countries, etc.). Recently, however, they have become one of the key strategies for mobile data offloading [?], whose main goal is to offload the traffic from cellular networks to other types of networks (e.g., WiFi infrastructured or MANET) in a synergic way, in order to address the overloading of the 3G/4G infrastructure.

In case of crowded environments (and thus dense networks) overloading may be even more critical, and opportunistic networking techniques can be usefully applied, as follows. Due to the typical Zipf-like shape of content interest, it is likely that large fractions of users in the crowd are interested in few, very popular contents (e.g., those mostly related to the area where the crowd gathers). Multicast can be a solution to reduce the traffic load only when content requests can be synchronised. When requests are generated dynamically by users, exploiting communications between users' devices is a more flexible solution, as content can be sent through the cellular network only to a few of them, exploiting opportunistic communications for the rest. The Device-to-Device technology addresses this goal to some extent, and is currently proposed in latest LTE releases. In this paper we focus on offloading through ad-hoc WiFi or Bluetooth technologies, as this approach permits to exploit additional portions of the spectrum (and, therefore, additional bandwidth) with respect to that allocated to cellular networks. A possible roadblock in this scenario is the fact that direct communications consume significant energy. To address this, nodes are typically operated in duty cycling mode, by letting their WiFi (or Bluetooth) interfaces ON only for a fraction of time. The joint effect of duty cycling and mobility is that, even if the network is dense, the resulting patterns in terms of communication opportunities is similar to that of conventional opportunistic networks, as devices are able to directly communicate with each other only when they come in one-hop radio range *and* both interfaces are ON.

The net effect of implementing a duty cycling scheme is thus the fact that some contacts between nodes are missed because the nodes are in power saving mode. Hence, detected intercontact times, defined as the time between two consecutive contact events during which a communication can take place for a pair of nodes, are longer than intercontact times determined only by mobility, when a duty-cycling policy is in place. This heavily affects the delay experienced by messages, since the main contribution to message delay is in fact due to the intercontact times. In our previous work [?], we have focused on exponentially distributed intercontact times and we have studied how these are modified by duty cycling, obtaining that intercontact times remain exponentially distributed but their rate is scaled by the inverse of the duty cycle (see Proposition 1, Section 3). Building upon this result, we have then investigated how the first moments of the end to end delay vary with the duty cycle for a number of opportunistic forwarding schemes. In

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addition, we have found that energy saving and end-to-end delay both scale linearly with the duty cycling, although the linear coefficient of scaling for the delay depends on the original distribution of the inter-contact times.

Our work in [1] assumed that the value of the duty cycle was given and studied its effects on important performance metrics such as the delay, the network lifetime, and the number of messages successfully delivered to their destination. More in general, the duty cycling can be seen as a parameter that can be configured, typically, based on some target performance metrics. To this aim, the main contribution of this paper is a mathematical model that allows us to tune the duty cycle in order to meet a given target performance, expressed as a probabilistic guarantee (denoted as p) on the delay experienced by messages. Considering probabilistic, instead of hard, guarantees, allows us to cover a very broad range of application scenarios also beyond best-effort cases – all but those requiring real-time streaming. Specifically, we study the case of exponential, hyper-exponential and hypo-exponential delays (please recall that any distribution falls into one of these three cases, at least approximately [2]), for original inter-contact times following an exponential or a Pareto distribution, which are two of the most relevant cases according to the analysis of real traces (see, e.g., [3]). In all these cases we derive the optimal duty cycle given a fixed maximum delay z and a given probabilistic guarantee on that maximum delay. For the simple case of exponential delays we are able to provide an exact solution. For the other cases, we derive an approximated solution and the conditions under which this approximation introduces a small fixed error ε (which is always below 0.14) on the target probability p. Specifically, in the worst case, the approximated duty cycle introduces an error on the target probability p of about 0.1 (hyper-exponential case) and 0.14 (hypo-exponential case), while in the other cases the error is well below these thresholds.

The paper is organised as follows. In Section 2 we overview the literature on duty cycle optimisation for opportunistic networks. After having introduced the network and duty cycle model that we consider in this work and recalled some preliminary results (Section 3), we provide an analytical model for the end-to-end delay in presence of duty cycling (Section 4). Then, in Section 5, we derive the optimal duty cycles for the case of exponential, hyper-exponential, and hypo-exponential delays. Finally, Section 6 concludes the paper.

2 Related work

There are not many contributions in the DTN literature studying the optimisation of the duty cycling policy. In [4], using a fixed duty cycle scheme, Wang et al. study the relationship between the probability of missing a contact and the associated energy consumption (considered inversely proportional to the contact probing interval). Building upon these results, [4] provides some heuristic algorithms to achieve an optimal contact probing. Differently from this work, in this paper we mathematically define the optimisation problem and we provide an analytical, closed form, result.

In [5], Gao and Li focus on the design of an adaptive duty cycle that minimises wakeups during intercontact times (which are useless, from a contact probing standpoint). Differently from [5], we have chosen to optimise the duty cycle directly, based on the performance goal that we want to achieve. While it is true that an optimisation based on intercontact times impacts directly on the delay performance, it is not straightforward how to control the one based on the other. With our model, instead, we can directly go from the requirements in term of probability of staying below a fixed delay threshold to the corresponding duty cycle value. In addition, differently from [5], we focus on a fixed duty cycle, similar to [6] [7] [4]. It is still an open research point which duty cycling strategy is to be preferred. However, preliminary results in [4] show that, under some assumptions, fixed duty cycle is the optimal strategy.

Another contribution focused on duty cycle optimisation is [8], in which Altman and Azad study the optimisation of node activation in DTN relying on a fluid approximation of the system dynamics. However, the problem analysed is different from the one studied in this paper, since in [8] nodes, once activated, remains active. In addition, this model is based on the assumption of i.i.d. intercontact times, while it has been shown that realistic intercontact times are intrinsically heterogeneous. For this reason, here we focus on heterogenous (but still independent) intercontact times.

3 Preliminaries

The duty cycle is for us any energy saving protocol that keep the device alternating between ON and OFF states in order to reduce the activity of the device and thus the battery consumption. In the ON state the device and all its interfaces are active, and thus it is able to detect contacts with other devices and to share the messages with them. Otherwise in the OFF state all the connectivity interfaces are switched off and the communication cannot be achieved. This simple abstract scheme is able to capture the most popular wireless technology (please see [9] for the details). In the literature are developed both deterministic and stochastic duty cycle policies, but in this work we will focus only on the deterministic one. However, as seen in [10], the deterministic case is able to capture the average behaviour of the stochastic case too. The main parameter of a deterministic duty cycle scheme is denoted with Δ defined as the ratio between the ON state and the total period of the duty cycle (that is the sum of the ON and the OFF state). It is well known that in opportunistic networks the delay experienced by the messages is mostly due to the time that each node has to wait before meeting another node, because the message reaches its final destination thought multiple exchanges between pair of nodes. This time is measured in the literature by the intercontact times. As the effect of the duty cycle is to reduce the number of contacts between nodes, the intercontact times obviously are changed by the the duty cycle. In [10] we have studied the effect of the duty cycle on the intercontact times, supposing to know the distribution of the intercontact times when no energy-saving protocols are applied in the network. In particular, when the intercontact times without duty cycle policies are exponential, the intercontact times are still exponential with rate scaled by the factor Δ. The exact result is the following:

Proposition 1. Considering a tagged pair of nodes i and j with exponential intercontact time of rate λ_{ij}, the detected intercontact time, i.e. the effective intercontact time when a duty cycling policy is in place, features approximately an exponential distribution with rate $\Delta\lambda_{ij}$, as long as $\lambda_{ij}T \ll 1$, where T is the duty cycling period.

When the intercontact times are distributed as a Pareto then the tail of the distribution is still Pareto with the same exponent and furthermore we can derive the first two moments of the detected intercontact times. Summing up, we have the following:

Proposition 2. Considering a tagged pair of nodes i and j with Pareto intercontact time S of rate λ_{ij} and exponent α_{ij} and supposing that $\frac{\lambda_{ij}T}{\alpha_{ij}-1} \ll 1$, where T is the duty cycling period, the detected intercontact time decays as a Pareto random variable with exponent α_{ij} and the first, the second and the coefficient of variation of the detected intercontact times $\tilde{S}$ are given by:

$$E[\tilde{S}] = \frac{1}{\Delta}E[S] \quad (1)$$

$$E[\tilde{S}^2] = \left(\frac{2}{\Delta} - 1\right)E[S]^2 + \frac{1}{\Delta}(E[S^2] + E[S]^2) \quad (2)$$

$$cv^2_{\tilde{S}} = \Delta\, cv^2_S + (1 - \Delta) \quad (3)$$

We have yet seen in [1] that the condition $\lambda_{ij}T \ll 1$ holds for the majority of contact traces available in the literature. For the Pareto case. The results cited above are obtained without considering the contact durations between nodes. The same analysis can be made using the results in [10] for contact duration non negligible, but as seen in the same work, the model with negligible contacts fits well the majority of the traces available in the literature.

4 The effect on the delay

In this section we exploit the results on the detected intercontact times summarized in section 3 in order to compute the first two moments of the delay for a set of representative forwarding strategies designed for opportunistic networks. The first step in this direction is to use the first two moments of the detected pairwise intercontact times for approximating the distribution of the intercontact time itself. Please note that while the analysis in the previous section focused on a tagged pair of nodes, in this section we study the whole network. So, when the intercontact times are exponentially distributed, denoting with their rates as λ_{ij} for node pair i, j, we have shown that, when assumption $\lambda_{ij}T \ll 1$ holds, the detected intercontact times follow approximately an exponential distribution. When the intercontact times are Pareto distributed, denoting with their rates as λ_{ij} and with their exponent as α_{ij} for node pair i, j, we have shown that, when assumption $\frac{\lambda_{ij}T}{\alpha_{ij}-1} \ll 1$ holds, the moments of the detected intercontact times change as in Proposition 2. Using this approximation, in the following we solve the analytical model proposed in [11] for both real intercontact times and detected intercontact times. The goal of this evaluation is to study how the first two moments of the delay are affected by energy saving techniques.

The forwarding model that we exploit represents the forwarding process in terms of a Continuous Time Markov Chain (CTMC) [12]. The chain has as many states as the nodes of the network and transitions between states depend both on the meeting process between nodes (i.e., their intercontact times) and on the forwarding protocol in use. Denoting the delay of messages from a generic node i to a tagged node d as D_{id}, and using standard Markov chain theory, it is possibile to derive the first two moments of D_{id} as in [11]. We report this result below for the convenience of the reader. Please note that this model (as most analytical models in the literature) assumes buffers and bandwidth large enough for accommodating all messages.

Lemma 1 (Delay's first and second moment). The first and second moment of the delay D_{id} for a message generated by node i and addressed to node d can be obtained from the minimal non-negative solutions, if they exists, to the following systems, respectively:

$$\forall i \neq d : E[D_{id}] = \frac{1}{\sum_{j\in\mathcal{R}_i}\lambda_{ij}} + \sum_{j\in\mathcal{R}_s-\{d\}} \frac{\lambda_{ij}}{\sum_{z\in\mathcal{R}_i}\lambda_{iz}} E[D_j]$$

$$\forall i \neq d :$$
$$E[(D_{id})^2] = \frac{2}{[\sum_{j\in\mathcal{R}_i}\lambda_{ij}]^2} + \sum_{j\in\mathcal{R}_s-\{d\}} \frac{\lambda_{ij}}{\sum_{z\in\mathcal{R}_i}\lambda_{iz}} E[D^2_{jd}] + $$
$$+2\sum_{j\in\mathcal{R}_s-\{d\}} \frac{\lambda_{ij}}{\sum_{z\in\mathcal{R}_i}\lambda_{iz}} \frac{1}{\sum_{j\in\mathcal{R}_i}\lambda_{ij}} E[D_{jd}]$$

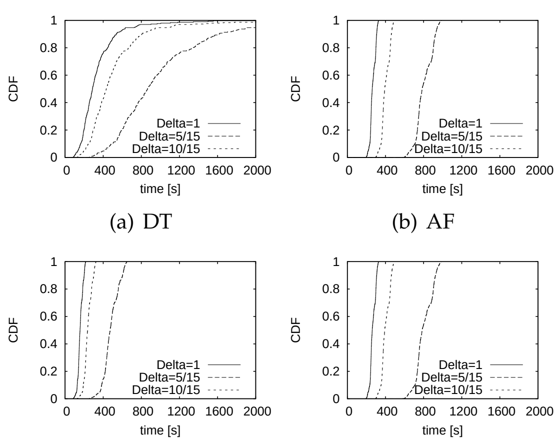
where we denote with $\mathcal{R}_i$ the set of possible relays towards destination d when the message is on node i (this set depends on the forwarding strategy in use).

In [11], we have defined a set of abstract policies able to capture significant aspects of popular state-of-the-art forwarding strategies. In the following we will focus on these policies. Under the Direct Transmission (DT) forwarding scheme, the source of the message is only allowed to hand it over to the destination itself, if ever encountered. With the Always Forward (AF) policy, the message is handed over by the source, and the following relays to the first nodes encountered. Both DT and AF are social-oblivious (also known as context-oblivious or randomized) policies, i.e., they do not exploit information on node social relationships and contact behavior. In [11] two social-aware policies were also defined. In social-aware policies, each intermediate forwarder hands over the message to nodes that have a higher probability of bringing the message closer to the destination, according to some predefined forwarding metrics. The first of these policies is Direct Acquaintance (DA), in which the forwarding metric is the contact rate with the destination ($\frac{1}{E[S_{id}]}$): a better forwarder is one with a higher contact rate with respect to the node currently holding the message. The second policy is Social Forwarding (SF), for which the forwarding metric is $\beta\frac{1}{E[S_{i,d}]} + (1-\beta)\sum_{j\in\mathcal{P}_i} w_{ij}\frac{1}{E[S_{jd}]}$, where $w_{ij} = \frac{\lambda_{ij}}{\sum_{j\in\mathcal{P}_i}\lambda_{ij}}$. With respect to the DA policy, which only captures direct meetings with the destination, SF is also able to detect indirect meetings, allowing nodes to select relays that not only meet the destination frequently but also meet nodes that meet the destination frequently.

In the rest of the paper we are going to indicate with D_Δ the random variable which represent the delay of the message when the duty cycle Δ is active and with D the random variable which represent the delay of the message when the duty cycle is not active (i.e. $D = D_1$). We now explore the effect of the duty cycle on the moments of the delay for the exponential intercontact times and for the Pareto intecontact times. We will see that the results are extremely different and in particular in the Pareto case, we will need to go deeper for finding the desired results.

4.1 The exponential case

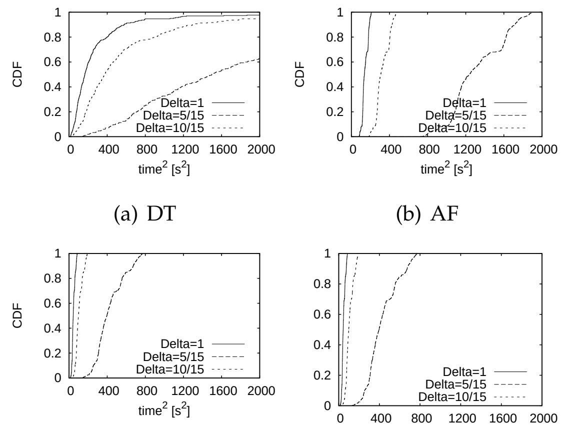
First of all we analyse the case of nodes intercontact times exponentially distributed. The fitting analysis presented in [3] has shown that contact rates in the traces follow a Gamma distribution. Below, we focus on the distribution parameters for the RollerNet scenario reported in [13], i.e., shape $\xi = 4.43$, rate $r = 1088$. We consider a network made up of 25 nodes and we solve the forwarding model described above in the case of duty cycle equal to $\frac{5}{15}$, $\frac{10}{15}$ and 1 (no duty cycling). Figures 1 and 2 show the CDF of the moments of the delay in this case. As expected, both the first and second moment become larger as we reduce the ON interval in the duty cycle. In fact, as discussed before, the net effect of duty cycling is to effectively reduce the number of usable contacts to only those happening during an ON period. The shorter the ON period, the fewer the usable contacts every T, the longer the delay.



(a) DT (b) AF (c) DA (d) SF

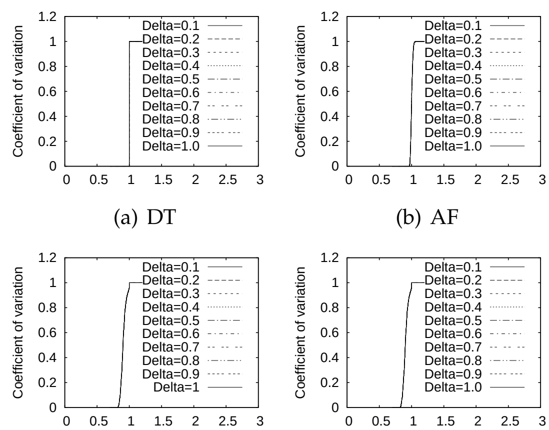
Fig. 1. CDF of the first moment of the delays for the different forwarding algorithms

Let us now see what happens to the coefficient of variation $c(D_\Delta)$ of the delay. From Figure 3 it can be seen that both $c(D_\Delta) < 1$ and $c(D_\Delta) \geq 1$ are possible. This means that the delay can be approximated with a hyper-exponential or a hypo-exponential distribution [2]. It is interesting to observe from Figure 3 that the coefficient of variation does not depend on the duty cycle Δ (in fact, all curves overlap). This means that, in the case of exponential intercontact times, the duty cycle does not affect



(a) DT (b) AF (c) DA (d) SF

Fig. 2. CDF of the second moment of the delays for the different forwarding algorithms



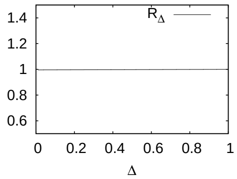
(a) DT (b) AF (c) DA (d) SF

Fig. 3. CDF of the coefficient of variation of the delay with different Δ

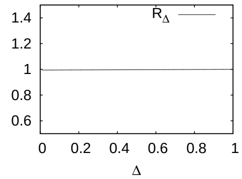
the variability of the delay experienced by messages, and thus for every duty cycle Δ, $c(D_\Delta) = c(D)$. As the coefficient of variation is defined as the square root of the ratio between the standard deviation and the mean value, i.e. $c(D_\Delta)^2 = \frac{\sigma(D_\Delta)}{E[\Delta]}$, if we are able to capture the dependence of the first moment of the delay $E[D_\Delta]$ from the duty cycle Δ, we immediately can derive the dependence of the second moment of the delay $E[D_\Delta^2]$ from Δ. We firstly tried with the inversion proportionality, i.e. with $E[D_\Delta] = \frac{E[D]}{\Delta}$. For testing this equation, in Figure 4 we plotted $\frac{E[D]}{\Delta E[D_\Delta]}$. Figure 4 (derived for the same parameters of the RollerNet scenario used above) tells us that the ratio stays around 1 independently of the specific duty cycle value Δ. This result is very interesting, because it shows that under exponential intercontact times the delay experienced by the message is scaled by the factor Δ. In summary, what we can derive is the following preposition:

Proposition 3. When the node intercontact times are exponentially distributed with rates λ_{ij} for node pair (i, j), that verify $\lambda_{ij} T \ll 1$, then:

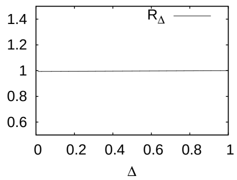
E1 The expected delay when a duty cycling policy is in place (denoted as $E[D_\Delta]$) is approximately equal to the



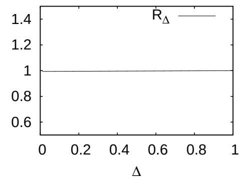
(a) DT



(b) AF



(c) DA



(d) SF

Fig. 4. $\frac{E[D]}{\Delta E[D_\Delta]}$ varying Δ in the different forwarding algorithms

expected delay $E[D]$ with no duty cycle scaled by a factor $\frac{1}{\Delta}$, i.e, $E[D_\Delta] = \frac{E[D]}{\Delta}$.

E2 The second moment of the delay when a duty cycling policy is in place (denoted as $E[D_\Delta^2]$) is approximately equal to the second moment of the delay $E[D^2]$ with no duty cycle scaled by a factor $\frac{1}{\Delta^2}$, i.e, $E[D_\Delta^2] = \frac{E[D^2]}{\Delta^2}$.

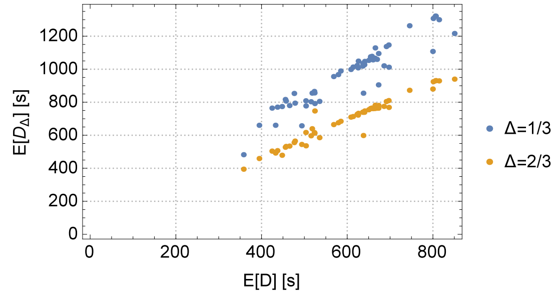
E3 The dependence of the coefficient of variation $c(D_\Delta)$ of the delay from Δ is negligible, i.e. $c(D_\Delta) = c(D)$.

Proof: E1 and E2 are the consequences of the analysis made above. The last easily derive by E1 and E2, using the definition of coefficient of variation. □

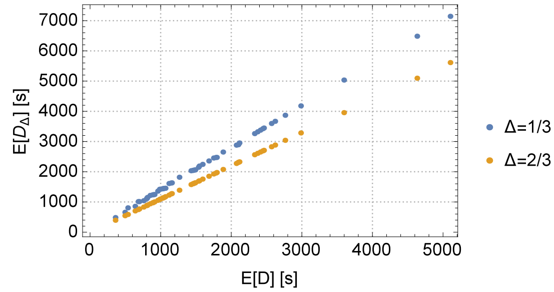
4.2 The Pareto case

In this section we want to study the case of a network in which the intercontact times between nodes are Pareto distributed, i.e. with PDF given by the function $f(x) = \frac{\alpha\, b^\alpha}{(b+x)^{\alpha+1}}$, for all $x > 0$. For this part we decided to analyse a network of nodes where the pairs intercontact times are Pareto distributed with parameters taken from a real trace. As we need finite first and second moments, we selected from the Rollernet trace the distributions of interontact times that are Pareto distributed with shape α bigger than 2. Then, we considered a network of 11 nodes and with intercontact times between the pairs of scale b equal to the 120s, the average scale of the selected intercontact times, and shape α randomly extract from the shapes of the selected intercontact times. Using different duty cycles, as made in Section 4.1, and then analysing the moments of the delay of the messages sent in the network, we discover that the relations E1-E3 valid for exponential intercontact times, are not working here. In figures 5 and 6 we can see that the dependences of the moments with duty cycle $E[D_\Delta], E[D_\Delta^2]$ from the respective moments without duty cycle $E[D], E[D^2]$ are linear. However, their slope are not going like $\frac{1}{\Delta}$ and $\frac{1}{\Delta^2}$ respectively.

From a deeper analysis, we discover that the two-order logarithm of the ratios $E[D_\Delta]/E[D]$ and of $E[D_\Delta^2]/E[D^2]$

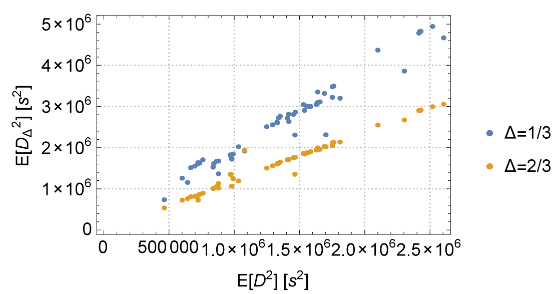


(a) DA

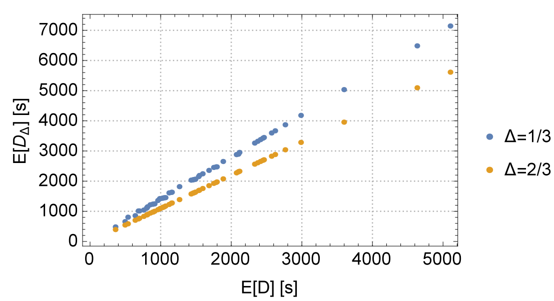


(b) DT

Fig. 5. Behaviour of $E[D_\Delta]$ respect to $E[D]$



(a) DA



(b) DT

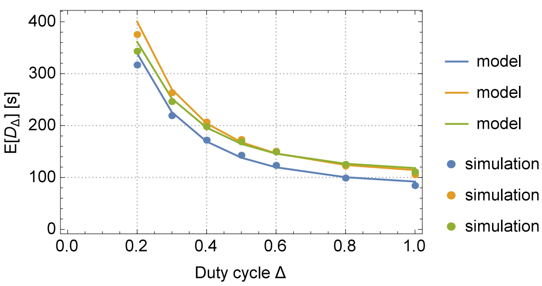
Fig. 6. Behaviour of $E[D_\Delta^2]$ respect to $E[D^2]$

are linearly dependent from Δ, and precisely for a general pair (i,j) of nodes, holds[1]:

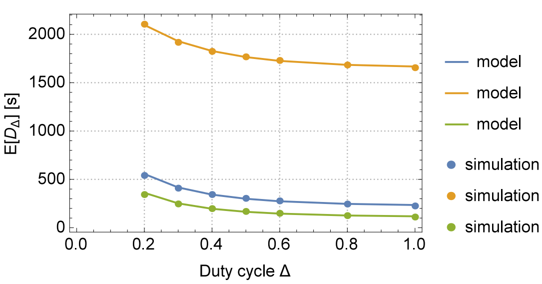
$$\log_2\left(\log\left(\frac{E[D_\Delta^{(i,j)}]}{E[D^{(i,j)}]}\right)\right) = m_1^{(i,j)}\Delta + q_1^{(i,j)}, \quad (4)$$

$$\log_2\left(\log\left(\frac{E[(D_\Delta^{(i,j)})^2]}{E[(D^{(i,j)})^2]}\right)\right) = m_2^{(i,j)}\Delta + q_2^{(i,j)}, \quad (5)$$

for some coefficients $m_1^{(i,j)}, q_1^{(i,j)}, m_2^{(i,j)}, q_2^{(i,j)} \in \mathbb{R}$.
The coefficients of equations (4) and (5) vary for each pair. Thus, using linear regression on the coefficients for all the different pairs we than obtain several functions that give the first (respectively the second) moment of the delay with a certain duty cycle, knowing the first (respectively the second) moment of the delay without duty cycle.



(a) DA

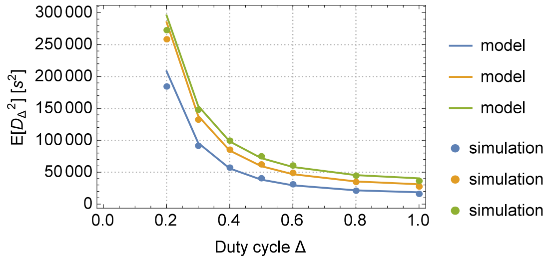


(b) DT

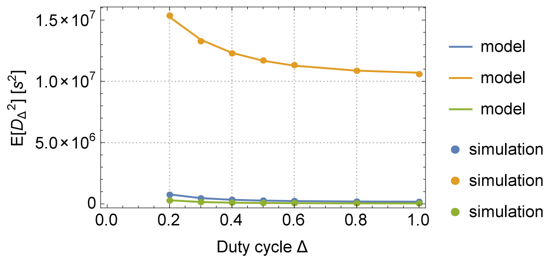
Fig. 7. First moment of the delay with different duty cycles for different forwarding algorithms

In Figure 7, 8 and 9, can be seen the comparison of the empirical data with result of the fitting of the first, the second and the coefficient of variation for a tagged pair of node with different duty cycles and forwarding policies. As can be seen, the function proposed is able to fit well the empirical data. As expected, the moments of the delay are larger when duty cycle is active. In Figure 9 can be seen the coefficient of variation of the delay. We can see here that the effect of the duty cycle is to reduce the coefficient of variation an in particular by applying a duty cycle, an hyper-exponential delay can become hypo-exponential (see for example Figure 9). We omitted the plots for the other pairs for lack of space, but the behaviour is the same. Summarizing, we have derived the following properties, which will be used extensively through the paper:

Proposition 4. When the node intercontact times are Pareto distributed, then the node pair (i,j) we have taht:

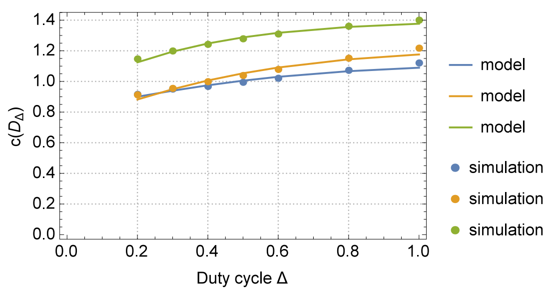


(a) DA

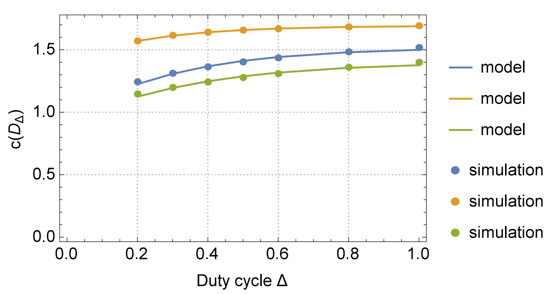


(b) DT

Fig. 8. Second moment of the delay with different duty cycles for different forwarding algorithms



(a) DA



(b) DT

Fig. 9. Coefficient of variation of the delay with different duty cycles for different forwarding algorithms

P1 The expected delay when a duty cycling policy is in place (denoted as $E[D_\Delta^{(i,j)}]$) is approximately given by:

$$E[D_\Delta^{(i,j)}] = \exp(\beta_1^{(i,j)}\, 2^{\gamma_1^{(i,j)}\Delta}) \cdot E[D^{(i,j)}].$$

P2 The second moment of the delay when a duty cycling policy is in place (denoted as $E[(D_\Delta^{(i,j)})^2]$) is approximately given by:

$$E\left[\left(D_\Delta^{(i,j)}\right)^2\right] = \exp(\beta_2^{(i,j)}\, 2^{\gamma_2^{(i,j)}\Delta}) \cdot E\left[\left(D^{(i,j)}\right)^2\right].$$

1. We here used a formulation with the logarithm to base 2 and e, but by the rules of changing base, the same formula is valid for other bases opportunely changing the coefficients m and q.

P3 The dependence of the coefficient of variation $c(D_\Delta^{(i,j)})$ of the delay from Δ is approximately given by:

$$c(D_\Delta^{(i,j)}) = \sqrt{\left(c(D^{(i,j)})+1\right)\frac{\exp\left(\beta_2^{(i,j)}2^{\gamma_2^{(i,j)}\Delta}\right)}{\exp\left(2\beta_1^{(i,j)}2^{\gamma_1^{(i,j)}\Delta}\right)}-1}.$$

Proof: P1 and P2 are the consequences of the analysis made above, and in particular the parameters are derived from 4 and 5 using $\gamma_k^{(i,j)} = m_k^{i,j)}$ and $\beta_k^{(i,j)} = 2^{q_k^{(i,j)}}$ for $k = 1, 2$. The last easily derive by P1 and P2, using the definition of coefficient of variation. □

5 Setting the duty cycle

In this section we are going to solve the problem of optimizing the duty cycle for each pair of nodes, so all the variables are referred to the node pair considered. For simplicity of notation, we here omit the superscript (i, j) of the variables which indicates the pair considered, and thus for example the delay of the message sent between the pair (i, j), will be denoted with D instead of $D^{(i,j)}$. In the following section, when we will analyse the whole network we will come back to the complete notation.

In this section we discuss how to derive the optimal duty cycle Δ_{opt} such that the delay of a tagged message remains, with a certain probability p, under a target fixed threshold z or, in mathematical notation, $\Delta_{opt} = \min\{\Delta : P\{D_\Delta < z\} \geq p\}$. Since the delay increases with Δ, the latter is equivalent to finding the solution to the following[2]:

$$\Delta_{opt} = \{\Delta : P\{D_\Delta < z\} = p\}. \tag{6}$$

In the following we will denote the CDF of D_Δ as $F_\Delta(x)$. In order to find the solution to Equation 6, the distribution of the delay D_Δ should be known. Although it is in general unfeasible to obtain an exact closed form for the distribution of D_Δ (except for some trivial cases, such as when the source node can only deliver the message to the destination directly), it is often possible to compute its moments, either exactly or approximately, under different distributions for intercontact times, as shown, e.g., in [14] [15]. When the first two moments of the delay can be derived, it is possible to approximate its distribution with either a hypo-exponential or hyper-exponential random variable, using the moment matching approximation technique [2]. So, exploiting properties derived in Section 4, according to the coefficient of variation c_Δ as $\sqrt{\frac{E[D^2]}{E[D]^2}-1}$, when c_Δ is greater than one, D_Δ can be approximated using a 2-stages hyper-exponential distribution with the same moments of D_Δ, as stated in the following Lemma.

Lemma 2 (Hyper-exponential approximation). The two moments matching approximation of D_Δ with coefficient of variation $c \geq 1$ is a 2-stages hyper-exponential distribution with parameters $(\lambda_1, p_1), (\lambda_2, p_2)$ given by the following:

$$\begin{cases} p_1 = \frac{1}{2}\left(1+\sqrt{\frac{c_\Delta^2-1}{c_\Delta^2+1}}\right) \\ \lambda_1 = \frac{2p_1}{E[D_\Delta]} \end{cases} \qquad \begin{cases} p_2 = 1-p_1 \\ \lambda_2 = \frac{2p_2}{E[D_\Delta]} \end{cases} \tag{7}$$

2. In the rest of the paper, for convenience of notation, we will drop subscript *opt* since all Δ we derive are the optimal ones.

Vice versa, when the coefficient of variation of the delay is smaller than 1 (but greater than $\frac{1}{\sqrt{2}}$ [16]), D_Δ can be approximated with an hypo-exponential distribution with CDF $F_X(x) = 1 - \frac{\mu_2}{\mu_2-\mu_1}e^{-\mu_1 x} + \frac{\mu_1}{\mu_2-\mu_1}e^{-\mu_2 x}$, for all $x \geq 0$, according to the following lemma.

Lemma 3 (Hypo-exponential approximation). The two moments matching approximation of D_Δ with a coefficient of variation. In the general case $c_\Delta < 1$, the two moments matching approximation of the hypoexponential is provided in [2]. However, with this approximation, μ_1 and μ_2 are such that they continuously oscillate as c_Δ increases. Hence, the techniques described in this paper cannot be exploited in this case. $c_\Delta \in (\frac{1}{\sqrt{2}}, 1)$ is an hypo-exponential distribution with rates μ_1, μ_2 given by the following:

$$\begin{cases} \mu_1 = \frac{2}{E[D_\Delta]} \cdot \frac{1}{1+\sqrt{1+2(c_\Delta^2-1)}} \\ \mu_2 = \frac{2}{E[D_\Delta]} \cdot \frac{1}{1-\sqrt{1+2(c_\Delta^2-1)}} \end{cases} \tag{8}$$

In the rest of the section, we will analyse the optimisation problem in Equation 6 assuming that D_Δ features an exponential (Section 4.1), hyper-exponential (Section 5.2) or hypo-exponential distribution (Section 5.3). Please note that all three cases are possible starting from exponential or Pareto intercontact times.

5.1 The exponential delay

If it was well defined, the simplest case would be when the delay features a coefficient of variation c equal to one. In this hypothesis, in fact, the distribution of the delay would be exponential and thus the solution would come out from the inversion of (6), that in this case is the following:

$$1 - e^{-\lambda_\Delta z} = p, \tag{9}$$

where the parameter is $\lambda_\Delta = E[D_\Delta]^{-1}$. Unfortunately, in the case of Pareto intercontact times, the coefficient of variation very with Δ and this means that if Δ^* is the solution of (9), the coefficient of variation $c(D_{\Delta^*})$ can be (and in general is) different from 0. For this reason, in this case the exponential distribution (and thus the solution of (9)) represents only a possible approximation of the distribution of the delay (respectively the solution of the problem). For the exponential intercontact times instead, the problem is well posed as the coefficient of variation c of the delay is independent from Δ and the equation (9) gives the exact solution when $c = 1$.

In both cases of exponential and Pareto intercontact times however, we are able to invert equation (6). Then, it is straightforward to derive Theorem 1 for exponential intercontact times and Theorem 2 for Pareto intercontact times.

Theorem 1 (Exponential intercontact times). If the intercontact times are exponentially distributed, one approximation of the optimal duty cycle using an exponential distribution for the delay D_Δ is given by the following:

$$\Delta = -\frac{\log(1-p)}{\lambda z}, \tag{10}$$

where we indicate with λ the parameter of the exponential distribution obtained with $\Delta = 1$, i.e., $\lambda = E[D]^{-1}$.

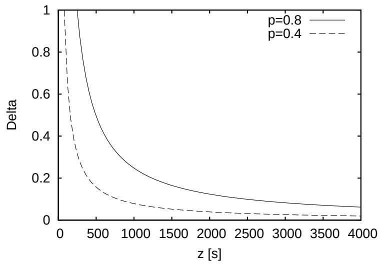


Fig. 10. Δ optimum for the exponential delay, varying the target delay threshold z.

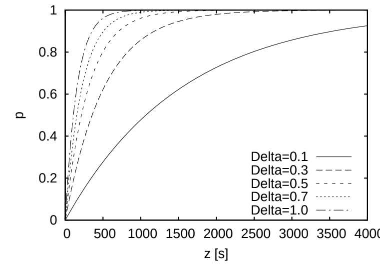


Fig. 11. Level set K_Δ for different values of Δ for the exponential delay, varying the target delay threshold z.

Proof: We know that $\lambda_\Delta = \frac{1}{E[D_\Delta]}$, hence, since $E[D_\Delta] \sim \frac{E[D]}{\Delta}$ (Property E1), we have that $\lambda_\Delta = \lambda\Delta$. Thus, we can rewrite Equation (6) as $1 - e^{-\lambda\Delta z} = p$, from which Δ can be easily obtained. □

Theorem 2 (Pareto intercontact times). If the intercontact times are Pareto distributed, the optimal duty cycle when D_Δ features an exponential distribution is given by the following:

$$\Delta = \frac{1}{\gamma}\log_2\left(\frac{\log\left(-\frac{z\lambda}{\log(1-p)}\right)}{\beta}\right), \tag{11}$$

where,as before, we indicate with λ the parameter of the exponential distribution obtained with $\Delta = 1$, i.e., $\lambda = E[D]^{-1}$ and with the parameters β and γ those of P1 obtained with the linear regression.

Proof: The proof works exactly as the proof of Theorem 1, but using properties P1 instead of E1 . □

For the sake of example, in Figure 10 we plot Δ obtained from Theorem 1 setting $p = 0.8$. $E[D]$ is set to $154s$, which is the average expected delay obtained in Section 5.1 for the policy DA. Figure 10 shows that, as expected, when the target delay threshold is too small, it is impossible to achieve it with a probabilistic guarantee p, regardless of the value of the duty cycle. Instead, starting from $z = -\frac{\log(1-p)}{\lambda}$, Δ is inversely proportional to z.

Varying the parameters z and p, Equation 6 describes a surface in $\mathbb{R}^3$, and more precisely the surface K given by:

$$K = \{(z,p,\Delta) \in \mathbb{R} \times [0,1] \times [0,1] : P\{D_\Delta < z\} = p\}. \tag{12}$$

Given a certain duty cycle $\Delta \in (0,1]$, we can thus describe K as the union of its level sets K_Δ or, in other terms, $K = \bigcup_{\Delta\in(0,1]} K_\Delta$ where:

$$K_\Delta = \{(z,p) \in \mathbb{R} \times [0,1] : P\{D_\Delta < z\} = p\}. \tag{13}$$

K_Δ is thus the set of pairs (z,p) that can be obtained with a given duty cycling Δ. It can be useful to plot K_Δ for different Δ in order to study whether it is possible to slightly compromise on the target performance in order to achieve a lower duty cycle. Assuming that we want $z = 250s$, in the exponential case (Figure 11) we can achieve it with a probability 0.8 with $\Delta = 1$ or with 0.68 with $\Delta = 0.7$, thus saving battery lifetime. Similarly, if we want to guarantee a target probability $p = 0.8$, with $\Delta = 1$ we obtain approximately $z = 250s$. If we are more flexible in terms of z, we can choose level set $K_{0.7}$ which gives $z = 350s$. This kind of analysis can be performed also for the hyper-exponential and hypo-exponential delays, with similar results.

5.2 The hyper-exponential delay

When the coefficient of variation of the delay is greater than one, the delay can be approximated with an hyper-exponential distribution as stated in Lemma 2. This means that Equation 6 becomes $1 - p_1e^{-\lambda_1 z} - p_2e^{-\lambda_2 z} = p$, where parameters $(\lambda_1, p_1), (\lambda_2, p_2)$ are given by Equation 7. In Sections 5.2.1 and 5.2.2 we are going to analyse the cases of exponential and Pareto intercontact times respectively.

5.2.1 The exponential intercontact times

From Equation (7), λ_1 and λ_2 depend on Δ (while p_1 and p_2 do not), thus, denoting with λ_1^0 and λ_2^0 the rates when $\Delta = 1$ and exploiting property P2, we can write Equation 6 as follows:

$$1 - p_1e^{-\lambda_1^0\Delta z} - p_2e^{-\lambda_2^0\Delta z} = p. \tag{14}$$

The exact solution Δ to this equation cannot be found analytically because Equation 14 cannot be inverted. However, in Theorem 3 below, we show how to obtain an approximated solution Δ_a that introduces a small error at most equal to ε.

Theorem 3 (Exponential intercontact times). Let us λ^0 denote $E[D]^{-1}$ and λ_1^0, λ_2^0 the rates of the hyper-exponential delay (Equation 7) for $\Delta = 1$. When delay D_Δ has coefficient of variation greater than one, given a threshold z of the delay and a target probability p, for every fixed $\varepsilon \geq \min\{\varepsilon_1, \varepsilon_2\}$ (whose definition is provided in the proof below), the duty cycle defined by:

$$\Delta_a = \begin{cases} \frac{1}{z}\Bigg[-\frac{1-p-p_2}{\lambda_2^0 p_2} + \\ \quad + \frac{1}{\lambda_1^0} W\left(\frac{p_1^2}{p_2^2} e^{\frac{\lambda_1^0(1-p-p_2)}{\lambda_2^0 p_2}}\right)\Bigg] & \text{if } \varepsilon_1 < \varepsilon_2 \\ -\frac{\log 1-p}{\lambda^0 z} & \text{if } \varepsilon_1 \geq \varepsilon_2, \end{cases} \tag{15}$$

where W is the Lambert function[3], verifies that $|F_{\Delta_a}(z) - p| \leq \varepsilon$ and so it is a good approximation of the solution to Equation 14.

Proof: We will provide below an intuitive sketch of the proof whose detailed version can be found in [17]. The idea for finding an approximate solution to Equation 14 is to identify an approximation $\tilde{F}(z)$ that is close to $F_\Delta(z)$ under some conditions. So, we build a function $\tilde{F}$ for which it is possible to solve Equation 14 and for which $\min\{\varepsilon_1, \varepsilon_2\}$

3. The Lambert function is defined as $W(x)e^{W(x)} = x$, for all $x \geq -\frac{1}{e}$

is the error introduced (we will clarify this point below). Specifically, we have identified the following function:

$$\tilde{F}(z) = \begin{cases} 1 - p_1 e^{-\lambda_1^0 \Delta z} - p_2(1 - \lambda_2^0 \Delta z) & \text{if } \varepsilon_1 < \varepsilon_2 \\ 1 - e^{-\lambda^0 \Delta z} & \text{if } \varepsilon_1 \geq \varepsilon_2 \end{cases} \tag{16}$$

Let us denote with $\tilde{F}_1(z)$ and $\tilde{F}_2(z)$ the two parts of $\tilde{F}(z)$ in the above equation. In $\tilde{F}_1(z)$, we have approximated the third term on the left hand side of Equation 14 using the Taylor expansion, after noting that this term contributes to $F_\Delta(z)$ less and less as the coefficient of variation c increases. Vice versa, the pure exponential behaviour ($\tilde{F}_2(z)$) dominates when c is close to 1. Both $\tilde{F}_1(z)$ and $\tilde{F}_2(z)$ can be solved to find Δ, from which Equation 15 follows.

The quality of these two approximations depends on the desired tolerance to the error that we inevitably introduce when we approximate $F_\Delta(z)$. If we tolerate a large error, either approximation can be chosen. Instead, if we want to achieve the smallest error, depending on the coefficient of variation of D_Δ we might have to prefer the one or the other. In the following we briefly discuss how to identify the minimum error introduced by $\tilde{F}_1(z)$ and $\tilde{F}_2(z)$, which we denote with ε_1 and ε_2 respectively. Let us start with $\tilde{F}_1(z)$. We want to find the region for which $|F_{\Delta_a}(z) - p| \leq \varepsilon$ or, equivalently, $|F_{\Delta_a}(z) - \tilde{F}_1^{(\Delta_a)}(z)| \leq \varepsilon$, where we denote with superscript (Δ_a) the fact that the CDF is computed using the approximated solution for Δ. Solving the above inequality, we find that it holds for all $p < p_{max}$, where p_{max} is a function of c and ε (due to lack of space, we do not report its formula here, please refer to [17] for details). Specifically, p_{max} monotonically increases with ε. So, if we want to derive the minimum error for which inequality $|F_{\Delta_a}(z) - p| \leq \varepsilon$ holds for all p, we have to solve equation $p_{max}(c, \varepsilon) = 1$. We obtain the following:

$$\varepsilon_1 = \frac{(a-1)\left(-(a-1)W\left(\frac{(a+1)^2 e^{\frac{a+1}{a-1}}}{(a-1)^2}\right) + a + 1\right)^2}{4(a+1)^2}, \tag{17}$$

where again $W(x)$ denotes the Lambert function and a is defined as $\sqrt{\frac{c^2-1}{c^2+1}}$.

Let us now consider $\tilde{F}_2(z)$. We are able to prove that function $|F_{\Delta_a}(z) - p|$ has a maximum in p^*. We derive p^* by finding the p in which the derivative of $|F_{\Delta_a}(z) - p|$ becomes zero. Then, ε_2 can be computed as $\varepsilon_2 = |F_{\Delta_a}(z) - p^*|$, obtaining the following:

$$\varepsilon_2 = \frac{1}{2}(a+1)^{-2/a}\left(a\sqrt{2-a^2}+1\right)^{1/a} \cdot \cdot \left(\frac{(a-1)(a+1)^2}{a\sqrt{2-a^2}+1} - \frac{a\sqrt{2-a^2}+1}{a+1} + 2\right), \tag{18}$$

where again $a = \sqrt{\frac{c^2-1}{c^2+1}}$. Thus, for both ε_1 and ε_2 we have derived a closed-form expression that tells us that the error that we make with our approximation is fixed for a given c. □

In Figure 12, we show how ε_1 and ε_2 vary with respect to the coefficient of variation c. As expected, for small c (recall that we are in the hyper-exponential case, so $c > 1$ by definition) the exponential assumption $\tilde{F}_2$ allows us to achieve smaller errors. The opposite is true for large c. The worst case is reached for $c \sim 1.5$, when the minimum error is around 0.1, which is still low. In Figure 13 we plot how the optimal duty cycle varies with z, setting the target probability to $p = 0.8$, for two values of coefficient of variation ($c = 1.3$ and $c = 3$). In both cases the approximation is good (the exact value is computed with standard numerical techniques to solve Equation 14). Specifically, when $c = 1.3$ the minimum error that can be achieved is 0.06 and is provided by $\tilde{F}_2(z)$, hence confirming the predominance of the exponential behaviour for c close to 1. Vice versa, when $c = 3$ the minimum error is 0.026 and is provided by $\tilde{F}_1(z)$. It is also interesting to notice that smaller duty cycles can be achieved when c increases, i.e., when the variability of the delay is higher.

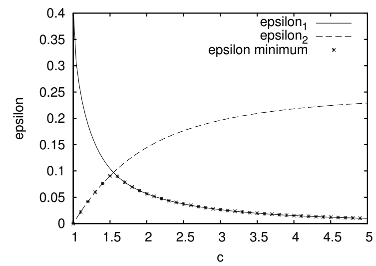


Fig. 12. Error introduced by $\tilde{F}_1(z)$ and $\tilde{F}_2(z)$, varying c.

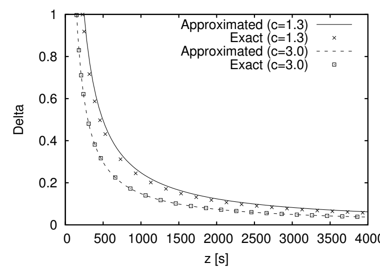


Fig. 13. Δ optimum (approximated vs exact) for the hyper-exponential delay, varying the target delay threshold z, when the target probability is $p = 0.8$ and $c \in \{1.3, 3\}$.

5.2.2 The Pareto intercontact times

When the intercontact times are Pareto distributed, the equation that has to be inverted becomes much more complicated than the one analysed in Section 5.2.1. In fact, as c_Δ depends from Δ, all the parameters of the distribution $p_1, p_2, \lambda_1, \lambda_2$ depend from Δ. Inverting this equation is not possible; however in the following theorem we describe an approximated solution and we provide an upper bound of the error ε.

Theorem 4 (Pareto intercontact times).
Let us indicate, as before, with λ the parameter of the exponential distribution obtained with $\Delta = 1$, i.e., $\lambda = E[D]^{-1}$ and with β and γ the parameters of P1 obtained with the linear regression. Consider then $p_1^a, p_2^a, \lambda_1^a, \lambda_2^a$ the parameters obtained from the following equations[4]:

$$\begin{cases} p_1^a = \frac{1}{2}\left(1 + \sqrt{\frac{c^2-1}{c^2+1}}\right) \\ \lambda_1^a = \frac{2p_1^a}{E[D_\Delta]} \end{cases} \quad \begin{cases} p_2^a = 1 - p_1^a \\ \lambda_2^a = \frac{2p_2^a}{E[D_\Delta]} \end{cases} \tag{19}$$

When delay D_Δ has coefficient of variation greater than one, given a threshold z of the delay and a target

4. We will explain in the proof how they are obtained.

probability p, for every fixed $\varepsilon \geq \min\{\varepsilon_1, \varepsilon_2\}$ (whose definition is provided in the proof below), the duty cycle defined by:

$$\Delta_a = \begin{cases} \frac{1}{\gamma \log(2)} \log\left[\frac{1}{\beta} \log\left(\frac{2p_1^a\, p_2^a\, z}{1-p-p_2^a+p_2^a\, W\left(-\frac{p_1^a}{p_2^a} e^{-(z-1+\frac{1-p}{p_2^a})}\right)} \right)\right] & \text{if } \varepsilon_1 < \varepsilon_2 \\ \frac{1}{\gamma} \log_2\left(\frac{\log\left(-\frac{z\lambda}{\log(1-p)}\right)}{\beta} \right) & \text{if } \varepsilon_1 \geq \varepsilon_2, \end{cases} \quad (20)$$

where W is the Lambert function, verifies that $|F_{\Delta_a}(z) - p| \leq \varepsilon$ and so it is a good approximation of the solution to Equation 14.

Proof: We illustrate a brief sketch of the proof. Let F be the CDF of the hyper exponential distribution. As previously said, one thing that makes this problem more difficult than the analogous for exponential intercontact times, is the dependence of the coefficient of variation $c(D_\Delta)$, and thus of the parameters p_1 and p_2 defined in equation (7), from Δ. The strategy that we used, consists in finding two different approximations of F, as in Theorem 3 and then choosing the ones which leads to the lower error. The first approximation is given by the function $\tilde{F}_1$ that can be obtained passing by two steps:

1) We approximate the parameters p_1 and p_2 with two other parameters p_1^a and p_2^a that are independent form Δ. Then, using those parameters and the correspondent λ_1^a and λ_2^a, we consider the hyper exponential distribution with CDF $\tilde{f}$.
2) We approximate the function $\tilde{f}$ with Taylor for finding the function $\tilde{F}_1$.

Let now start with 1. By P3, we have that $c(D_\Delta) = \sqrt{\kappa(\Delta)\,(c^2+1) - 1}$, where:

$$k(\Delta) = \exp\left(\beta_2 2^{\gamma_2 \Delta} - \beta_1 2^{\gamma_1 \Delta + 1}\right). \quad (21)$$

As the parameters $\beta_1, \beta_2, \gamma_1, \gamma_1$ are obtained by analysing experimental data, we are not able to fully characterize the function $\kappa(\Delta)$. However, in our all experimental data, we saw that $\kappa(\Delta) \leq 1$ and that is close to 1. We can observe that the first fact means that $c(D_\Delta) < c$, i.e. the duty cycle reduces the coefficient of variation. On the other side, we use the closeness to 1, for approximating $c(D_\Delta) \simeq c$. Thus the approximated parameters becomes those of equation (19). The function $\tilde{f}$ is then given by:

$$\tilde{f}(z) = 1 - p_1^a e^{-\lambda_1^a z} - p_2^a e^{-\lambda_2^a z}. \quad (22)$$

For the approximation in 2, we apply Taylor to the function $\tilde{f}$ as in the proof of Theorem 3. Thus $\tilde{F}_1$ is:

$$\tilde{F}_1(z) = 1 - p_1^a e^{-\lambda_1^a z} - p_2^a (1 - \lambda_2^a z). \quad (23)$$

As second approximation for the function F, we choose the exponential distribution i.e. the following CDF:

$$\tilde{F}_2(z) = 1 - e^{-z/E[D_\Delta]}. \quad (24)$$

Using $\tilde{F}_1$ and $\tilde{F}_2$ instead of F in equation (14), the identity becomes invertible and the solutions are the two pieces of equation (20), respectively Δ_a^1 and Δ_a^2. Obviously we choose the one or the other depending on which from $\tilde{F}_1$ or $\tilde{F}_2$ best approximate the CDF F. We define ε_1 and ε_2 as the distance of solution from the target probability threshold p chosen. In other words, for $i = 1, 2$,

$$\varepsilon_i(z, p) = |P(\Delta_a^i) - p|, \quad (25)$$

that depend only from the thresholds and the others parameters of the problem. This concludes the proof. □

5.3 The hypo-exponential case

When the coefficient of variation c of the delay D_Δ is smaller than one, following Lemma 3, it is possible to approximate the delay with a hypo-exponential distribution. In particular, using property P2, if we denote with μ_1^0 and μ_2^0 the parameters obtained when $\Delta = 1$ in Equation 8, we can rewrite Equation 6 making explicit the dependence on Δ:

$$1 - \frac{\mu_2^0}{\mu_2^0 - \mu_1^0} e^{-\mu_1^0 \Delta z} + \frac{\mu_1^0}{\mu_2^0 - \mu_1^0} e^{-\mu_2^0 \Delta z} = p. \quad (26)$$

As in the hyper-exponential case, this equation can not be directly inverted for finding Δ, but it is possible to derive an approximate solution for which a small fixed (for a given c) error is introduced.

Theorem 5. Let μ_1^0 and μ_2^0 be the parameters given by Equation 8 with $\Delta = 1$. When the delay D_Δ has coefficient of variation smaller than one, the duty cycle defined by:

$$\Delta_a = \begin{cases} -\frac{1}{\mu_1^0 z} \log\left[(1-p) \cdot \frac{\mu_2^0 - \mu_1^0}{\mu_2^0}\right] & \text{if } \varepsilon_1 < \varepsilon_2 \\ -\frac{\log 1-p}{\lambda^0 z} & \text{if } \varepsilon_1 \geq \varepsilon_2, \end{cases} \quad (27)$$

verifies that $|F_{\Delta_a}(z) - p| \leq \varepsilon$ (with $\varepsilon \geq \min\{\varepsilon_1, \varepsilon_2\}$, see the proof in [17]), and so it is a good approximation of the solution to Equation 26.

Due to lack of space and since the rationale follows that of the proof for Theorem 3, we omit the proof of the above theorem, which can however be found in [17].

In Figure 14 we plot ε_1 and ε_2 varying c. When c is close to one, both approximations are very good. For values of c roughly in the interval $(0.83, 0.97)$, $\tilde{F}_1(z)$ provides better results, while, for low values of c, $\tilde{F}_2(z)$ is to be preferred. In Figures 15(a) and 15(b) we show how the optimal duty cycle varies with z, setting the target probability to $p = 0.8$, for two values of coefficient of variation ($c = 0.75$ and $c = 0.9$, respectively). In both cases the approximation and the exact value are very close. In Figure 15(a) the minimum error that can be achieved is 0.13 and is provided by $\tilde{F}_2(z)$, while in Figure 15(b) the minimum error is 0.05 and is provided by $\tilde{F}_1(z)$.

6 Conclusion

In this work we have studied how to optimise the duty cycle in order to guarantee, with probability p, that the delay of messages remains below a threshold z, assuming that intercontact times are either exponentially or Pareto distributed. We have provided an exact solution for the case in which the delay follows an exponential distribution, and approximated solutions for the cases in which the coefficient of variation of the delay is greater than or smaller than 1. We have also demonstrated that the approximation of Δ

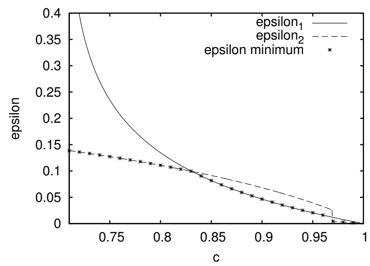
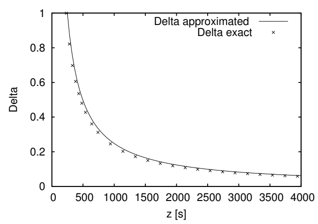
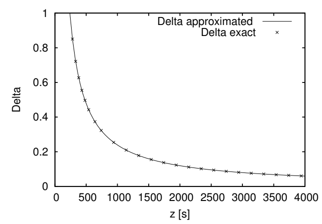


Fig. 14. Error introduced by $\tilde{F}_1(z)$ and $\tilde{F}_2(z)$, varying c.



(a) $c = 0.75$



(b) $c = 0.9$

Fig. 15. Δ optimum (approximated vs exact) for the hypo-exponential delay, varying the target delay threshold z, with target probability $p = 0.8$ and $c \in \{0.75, 0.9\}$.

introduces an error ε whose formula we have provided and that is small and fixed for a given coefficient of variation c of the delay, in the case of exponential intercontact times.

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