# The Critical Transmitting Range for Connectivity in Mobile Ad Hoc Networks

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**Abstract**

In this paper we study the critical transmitting range (CTR) for connectivity in mobile ad hoc networks. We prove that $r_{\mathcal{M}} = c\sqrt{\frac{\ln n}{\pi n}}$ for some constant $c \geq 1$, where $r_{\mathcal{M}}$ is the CTR in presence of $\mathcal{M}$-like node mobility and $n$ is the number of network nodes. Our result holds for an arbitrary mobility model $\mathcal{M}$ such that: (1) $\mathcal{M}$ is obstacle free, and (2) nodes are allowed to move only within a certain bounded area.

We also investigate in detail the case of random waypoint mobility, which is the most common mobility model used in the simulation of ad hoc networks. Denoting with $r_p^w$ the CTR with random waypoint mobility when the pause time is set to $p$ and node velocity is set to $v$, we prove that $r_p^w = \frac{p+\frac{0.521405}{v}}{p}\sqrt{\frac{\ln n}{\pi n}}$ if $p > 0$, and that $r_0^w \gg \sqrt{\frac{\ln n}{n}}$. The results of our simulations also suggest that if $n$ is large enough ($n \geq 50$), $r_0^w$ is well approximated by $\frac{r}{4}\ln n$, where $r$ is the critical range in case of uniformly distributed nodes.

The results presented in this paper provide a better understanding of the behavior of a fundamental network parameter in presence of mobility, and can be used to improve the accuracy of mobile ad hoc network simulations.

*Keywords*: Critical transmitting range, connectivity, random waypoint model, mobility modeling, ad hoc networks.

# 1 Introduction

One of the prominent features of ad hoc networks is mobility. As actual implementations of medium to large scale ad hoc networks are scarce to date, the performance of ad hoc networking protocols are evaluated primarily through simulation. Due to the lack of real world movement patterns, mobility is usually simulated by implementing synthetic mobility models. Several such models have been proposed in the literature, modeling both individual and group movement [6, 13, 14]. For a survey on mobility models for ad hoc networks, the reader is referred to [2, 9].

Several recent papers have investigated the accuracy of ad hoc network simulation in presence of node mobility. In particular, two properties of mobile networks that could affect simulation accuracy have been identified: the *border effect* and, more recently, the *speed decay* phenomenon.

The border effect, which was first noticed in [4] and investigated in detail in [3, 5, 8, 21, 22], may occur when node mobility is constrained within a limited area. Depending on the mobility model and/or the mobility parameters, the presence of the border may bias the long-term node spatial distribution in presence of mobility, which is no longer uniform. If this occurs, the accuracy of mobile networks simulation could be impaired: in fact, the initial node distribution, which is usually uniform, is different from the long-term distribution, which is not uniform.

The speed decay phenomenon, which was first noticed in [25] and further investigated in [26], occurs when nodes' velocities are chosen at random in a certain interval $[v_{min}, v_{max}]$. If $v_{min} < v_{max}$, the asymptotic average node velocity is different from the average node velocity at the beginning of the simulation. This phenomenon becomes critical when $v_{min} = 0$ (as it is often assumed when simulating ad hoc networks), since in this case the mobile system will eventually converge to a stationary one.

Due to the combination of the border effect and the speed decay phenomenon, the results obtained when simulating mobile networks could be quite inaccurate, unless the simulation methodology is carefully chosen. This is especially true in case of random-waypoint mobility (RWP) [14], which is the most common mobility model used in the literature. In this model, nodes are initially distributed in a (usually bi-dimensional) region $R$. Then, every node chooses uniformly at random a destination in $R$, and moves towards it along a straight line, with a velocity chosen uniformly at random in the interval $[v_{min}, v_{max}]$. When the node reaches the destination, it remains stationary for a predefined pause time $p$, and then it starts moving again according to the same rule. It is known that the RWP model suffers of both the border effect [2, 4, 8] and the speed decay phenomenon [25, 26]. Thus, the performance of RWP mobile networks should be evaluated only after a certain "warm-up" period, which must be sufficient for the system to reach the "steady state" in terms of both node spatial distribution and average node velocity. If the system performance is evaluated during the "warm-up" period, the obtained results might be quite misleading. For instance, in [25] it is shown that the inaccuracy can be as high as 40% when evaluating certain ad hoc routing metrics.

In this paper, we make a step forward toward the definition of a methodology for accurate simulation of mobile networks. We consider the *critical transmitting range* (CTR) for connectivity, and we study how this network metric changes in presence of node mobility. The CTR corresponds to the minimum *common* value of the nodes' transmitting range that produces a connected communication graph[1]. It is known that setting the nodes' transmitting range to the critical value minimizes energy consumption while maximizing network capacity [12, 15].

Due to the possible occurrence of the border effect, the CTR in presence of mobility is in general different from the critical transmitting range in the stationary case with uniformly distributed nodes. This observation discloses another potential source of inaccuracy in the simulation of mobile networks. Suppose we want to evaluate the performance of a routing protocol for mobile networks. The main effects of mobility on a routing protocol are: ($i$) frequent route reconfigurations and ($ii$) occasional network disconnections. In order to fully understand the behavior of the protocol, the relative effects of ($i$) and ($ii$) on the routing performance should be carefully evaluated. It is clear that the frequency of network disconnections depends on the choice of the nodes' transmitting range: the larger

[1]In this paper, we assume that nodes use omni-directional antennas, and that any two nodes $u$ and $v$ can communicate through a bi-directional wireless link if and only if $u$ is within $v$'s range, and $v$ is within $u$'s range.

the range, the less likely it is that the network becomes disconnected. On the other hand, for the reasons described above (energy consumption and network capacity) the nodes' transmitting range cannot be excessively large. Thus, setting the transmitting range to the critical value for connectivity is a reasonable choice. However, a wrong setting of the CTR might lead to an incorrect interpretation of the simulation results. For instance, if the CTR in presence of mobility is larger than in case of stationary networks[2] and the CTR is wrongly set as if the network were stationary, then there is a relatively high likelihood of generating a disconnected topology as the nodes move. In turn, this causes a relatively low packet delivery rate, which could erroneously be interpreted as a scarce protocol's ability of performing route maintenance (which might not be the case).

The example above outlines the importance of a correct estimation of the CTR when simulating mobile ad hoc networks. In this paper, we investigate the relation between the CTR in case of stationary networks with uniformly distributed nodes and the CTR in presence of $\mathcal{M}$-like node mobility, where $\mathcal{M}$ is an arbitrary bounded and obstacle free mobility model (see Section 4 for the definition). Denoting with $r$ and $r_{\mathcal{M}}$ the critical range in case of stationary and mobile networks, and denoting with $n$ the number of network nodes, we prove that $\lim_{n\to\infty} \frac{r_{\mathcal{M}}}{r} = c$ for some constant $c \geq 1$.

We also investigate in detail the case of RWP mobility, which is the most common mobility model used in the simulation of ad hoc networks. Denoting with $r_p^w$ the CTR with random waypoint mobility when the pause time is set to $p$ and node velocity is $v_{min} = v_{max} = v$, we prove that $r_p^w = \frac{p + \frac{0.521405}{v}}{p}\sqrt{\frac{\ln n}{\pi n}}$ if $p > 0$, and that $r_0^w \gg \sqrt{\frac{\ln n}{n}}$ otherwise. Besides studying the asymptotic properties of the CTR, we also give a formula

[2]As proved in this paper, this is actually the case, at least for bounded, obstacle free mobility.

that can be used to derive a good estimate of the CTR in presence of mobility given the critical range for stationary networks. Based on the results of extensive simulations, we argue that if $n$ is large enough ($n \geq 50$), $r_0^w$ is well approximated by $\frac{r}{4} \ln n$. The error obtained using this formula is below 6.4%. Since $r_p^w$ when $p > 0$ is smaller than $r_0^w$, our formula provides an upper bound to the CTR is presence of arbitrary RWP mobility.

The rest of this paper is organized as follows. In Section 2 we overview some related work on the characterization of the CTR. In Section 3, we introduce a result taken from the theory of geometric random graphs, which we use in the remainder of the paper. In Section 4, we provide a partial characterization the CTR of mobile networks, and a more accurate characterization for the case of RWP mobility. This latter characterization is qualitatively confirmed by the simulation results presented in Section 5. Section 6 concludes.

## 2 Related work

The critical transmitting range for connectivity has been investigated in several papers, mostly under the assumption of uniform node distribution.

In [11], $R$ is the disk of unit area. The authors show that if the units' transmitting range is set to $r = \sqrt{\frac{\ln n + c(n)}{\pi n}}$, then the resulting network is connected with high probability (i.e., with probability 1 as $n$ grows to infinity) if and only if $c(n) \to +\infty$. The minimum value of $r$ which ensures connectivity with high probability is called the *critical transmitting range*. In [16], the authors obtain a similar expression for the critical transmitting range when nodes are distributed in the unit square $[0, 1]^2$, which is shown to be

$r = c\sqrt{\frac{\ln n}{n}}$, for some constant $c > 0$. The CTR for $k$-connectivity has been investigated in [1].

The results above assume that the deployment region $R$ is fixed, and investigate the asymptotic behavior of $r$ as $n$ grows to infinity, i.e. for increasing node density. This means that the results of [1, 11, 16] in principle can be used only for very dense networks. On the other hand, it is known that real ad hoc networks cannot be too dense, due to the problem of spatial reuse: when a node is receiving a message, all the nodes within its interference range must be silent, in order not to corrupt the transmission. If the node density is very high, many nodes must remain silent when a node is receiving, and the overall network capacity is compromised [12]. In order to circumvent this problem, we can let the area $a$ of the deployment region $R$ be a further parameter, and characterize the critical transmitting range as $a \rightarrow +\infty$. Necessary and sufficient conditions for asymptotic connectivity in this model have been obtained in [24].

Although interesting, the results above are valid only for *stationary* wireless networks. To the best of our knowledge, the only analytical investigation of the CTR in presence of mobility is the one reported in [7], where a lower bound to the CTR is computed through numerical integration. However, no explicit formula of the CTR in presence of RWP mobility is given. Other papers have investigated the CTR in presence of mobility by means of simulation [23, 24]: for instance, in [24] it is shown that a moderate increase in the transmitting range (in the order of 25%) with respect to the stationary case is sufficient to provide good connectivity in presence of mobility.

# 3 Geometric random graphs

The theory of geometric random graphs (GRG) has been often used in the derivation of analytical characterizations of the CTR.

In the theory of GRG, a set of points is distributed according to some probability density function (pdf) in a $d$-dimensional region, and some property of the resulting node placement is investigated. For example, the longest nearest neighbor link, the longest edge of the Euclidean Minimum Spanning Tree (MST), and the total cost of the MST have been investigated. For a survey of GRG, the reader is referred to [10].

Some of these results can be applied in the study of connectivity in ad hoc networks. For instance, consider a set $S$ of points distributed in the deployment region $R$. It is known that the minimum common value of the transmitting range such that the resulting communication graph is connected equals the length of the longest edge of the Euclidean Minimum Spanning Tree built on $S$ [19]. Hence, results concerning the asymptotic distribution of the longest MST edge [18, 19] can be used to characterize the critical transmitting range, as it has been done by the authors of [16]. Another notable result of the theory of GRG is that, under the assumption of uniformly distributed points, the longest nearest neighbor link and the longest MST edge have the same value (asymptotically). In terms of the resulting communication graph, this means that connectivity occurs (asymptotically) when the last isolated node disappears from the graph. This observation can be generalized to the case of $k$-connectivity: when the minimum node degree becomes $k$, the graph becomes $k$-connected [20]. This result has been used in [1] to characterize the $k$-connectivity of ad hoc networks.

In the next section, we will use the following result due to Penrose [18], which characterizes the distribution of the longest MST edge for points distributed according to an arbitrary pdf with connected and compact support[3]:

**Theorem 1 (Penrose 1999).** *Let* $X_1, X_2, X_3 \dots$ *be independent random points in* $\mathbb{R}^2$*, and assume that the points are distributed according to a common pdf* $f$*, having connected and compact support* $\Omega$ *with smooth boundary* $\partial\Omega$*. Further, assume that* $f$ *is continuous on* $\partial\Omega$*. Let* $M_n$ *denote the length of the longest MST edge built on the first* $n$ *points of this random process. Then:*

$$\lim_{n \to \infty} \frac{n\pi \left(M_n\right)^2}{\ln n} = \frac{1}{\min_{\Omega} f},$$

*almost surely.*

We recall that the boundary $\partial\Omega$ is smooth if and only if it is twice differentiable (technically, if it is in $C^2$).

Theorem 1 holds in the hypothesis that $\min_{\Omega} f > 0$. However, Penrose states that, given the similarities with the result on the largest nearest neighbor link of [17], the theorem holds also when $\min_{\Omega} f = 0$ (up to details which are not written in the paper). In this case, the value of the limit must be intended as $+\infty$.

In words, Theorem 1 states that the asymptotic behavior of the longest MST edge (and, consequently, of the critical transmitting range) depends only on the *minimum value* of the pdf used to distribute the nodes in $\Omega$. In the next Section, we use this result to characterize the asymptotic behavior of the CTR in presence of mobility.

[3]We recall that the support of a pdf is the set of points in which it has non-zero value.

# 4 The critical transmitting range of mobile networks

Our first result is the characterization of the CTR in presence of *bounded* and *obstacle free* mobility, which we now define.

**Definition 1.** *Let $R$ be a bounded region, and let $\partial R$ be its boundary. Let $\mathcal{M}$ be an arbitrary mobility model, and let $f_{\mathcal{M}}$ be the pdf that resembles the long-term node spatial distribution generated by $\mathcal{M}$-like mobility. $\mathcal{M}$ is bounded within $R$ if the support of $f_{\mathcal{M}}$ is contained in $R$. $\mathcal{M}$ is obstacle free if the support of $f_{\mathcal{M}}$ contains $R - \partial R$.*

In words, a mobility model is bounded within $R$ if the nodes are allowed to move only within that region, while it is obstacle free if the probability of finding a mobile node in any sub-region of $R$ (excluding the border) is greater than 0.

Note that most of the mobility models used in the simulation of ad hoc networks are bounded (provided a suitable border rule is implemented [4]) and obstacle free: for instance, the random waypoint model, the random direction model, and the Brownian-like model are bounded and obstacle free.

For simplicity, in the rest of this paper we assume that $R = [0,1]^2$, i.e., it is the unit square.

**Theorem 2.** *Let $\mathcal{M}$ be an arbitrary mobility model which is bounded within $R = [0,1]^2$ and obstacle free. Furthermore, assume that $f_{\mathcal{M}}$ is continuous on $\partial R$, and $\min_R f_{\mathcal{M}} > 0$. The critical transmitting range for connectivity of an ad hoc network with $\mathcal{M}$-like mobility is $r_{\mathcal{M}} = c\sqrt{\frac{\ln n}{\pi n}}$ with high probability, for some constant $c \geq 1$.*

*Proof.* We observe that, if the hypotheses of Penrose's theorem hold, our result follows

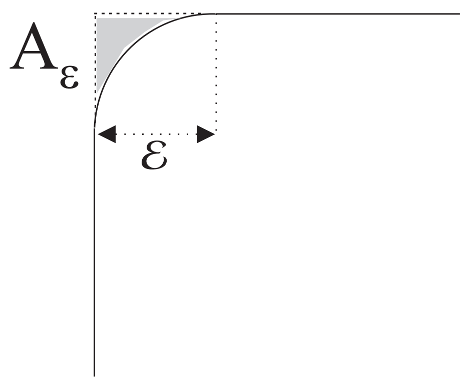


Figure 1: The corner of $\partial R$ is rounded using a portion of the circle of radius $\varepsilon$. For each corner, the density residing in the shaded area $A_\varepsilon$ is uniformly re-distributed in $R_\varepsilon$.

immediately since $\min_R f_{\mathcal{M}} > 0$ by hypothesis. Thus, we only have to show that the hypotheses of Penrose's theorem are satisfied.

First, we observe that since $\mathcal{M}$ is bounded within $R$ and obstacle free, the support of $f_{\mathcal{M}}$ is contained in $R$ and contains $R - \partial R$. Since $R$ is connected and compact, it follows that the support of $f_{\mathcal{M}}$ is connected and compact also. Furthermore, $f_{\mathcal{M}}$ is continuous on $\partial R$ by hypothesis. The only hypothesis left to prove is that $\partial R$ is smooth. Unfortunately, this is not true due to the presence of the corners. This problem can be circumvented by considering the region $R_\varepsilon$, obtained by "rounding" the corners of $\partial R$ with a portion of the circle of radius $\varepsilon$ (see Figure 1). The boundary of $R_\varepsilon$ is smooth for any value of $0<\varepsilon<1/2$, and $\lim_{\varepsilon\to 0} R_\varepsilon = R$. In order to preserve the probability measure when considering the restricted region $R_\varepsilon$, the fraction of the original density $f_{\mathcal{M}}$ residing in the areas such as area $A_\varepsilon$ in Figure 1 is uniformly re-distributed in $R_\varepsilon$. Denoting with $f^\varepsilon_{\mathcal{M}}$ this modified pdf, it is immediate to see that $\min_R f_{\mathcal{M}} \leq \min_{R_\varepsilon} f^\varepsilon_{\mathcal{M}}$, $\lim_{\varepsilon\to 0} (\min_{R_\varepsilon} f^\varepsilon_{\mathcal{M}}) = \min_R f_{\mathcal{M}}$, and $\lim_{\varepsilon\to 0} f^\varepsilon_{\mathcal{M}} = f_{\mathcal{M}}$. By Theorem 1 we can write:

$$\lim_{n \to \infty,\ \varepsilon \to 0} \frac{n\pi \left(M_n^{(\varepsilon)}\right)^2}{\ln n} = \lim_{n \to \infty} \frac{n\pi \left(M_n\right)^2}{\ln n} = \frac{1}{\min_R f_{\mathcal{M}}} = c \geq 1 \ ,$$

almost surely, where $M_n^{(\varepsilon)}$ denotes the longest MST edge (hence, the critical transmitting range) when nodes are distributed according to $f_{\mathcal{M}}^{\varepsilon}$ on $R_\varepsilon$. The fact that $c \geq 1$ follows by observing that, since the area of $R$ is 1 and $f_{\mathcal{M}}$ is a pdf with support that contains $R - \partial R$ and is contained in $R$ (i.e., $\int_R f_{\mathcal{M}} = 1$), the minimum value of $f_{\mathcal{M}}$ on $R$ is at most equal to 1. This concludes the proof of the theorem. □

**Corollary 1.** *Let $r$ denote the CTR for connectivity in stationary (uniformly distributed) networks, and let $r_{\mathcal{M}}$ denote the same range in presence of $\mathcal{M}$-like mobility, where $\mathcal{M}$ satisfies the hypotheses of Theorem 2. Then, $\lim_{n \to \infty} \frac{r_{\mathcal{M}}}{r} = c$, for some constant $c \geq 1$.*

*Proof.* Denoting by $f_u$ the pdf that distributes uniformly at random the nodes in $R$, we have $\min_R f_u = 1$. As observed in the proof of Theorem 2, the minimum value on $R$ of any other pdf with support contained in $R$ is at most 1, and the corollary is proved. □

In words, corollary 1 states that *every bounded and obstacle free type of mobility is detrimental for network connectivity*, since the CTR can only increase. Note that this result is asymptotic, so it is true only for very large networks. If the network is small, the situation might be even reversed (see the simulation results reported in the next section).

The characterization of the CTR for mobile networks stated in Theorem 2 is only partial, since it leaves the value of the constant $c$ unspecified. A more accurate characterization can be given only if the actual formula of $f_{\mathcal{M}}$ is known. To the best of our knowledge, the only mobility model for which an accurate approximation of the pdf that models the long-term node spatial distribution has been derived is the RWP model. This approximation is given in [3].[4] The pdf is the following:

**Theorem 3 (BettstetterRestaSanti 2003).** *The asymptotic spatial density function of a node moving in $R = [0,1]^2$ according to the random waypoint model of with pause time $p$ and $v_{min} = v_{max} = v$, is accurately approximated by*

$$f_{RW}(x,y) = \begin{cases} P_p + (1-P_p) f_m(x,y) & \text{if}(x,y) \in [0,1]^2 \\ 0 & \text{otherwise} \end{cases},$$

*where $P_p = \frac{p}{p+\frac{0.521405}{v}}$ and*

$$f_m(x,y) = \begin{cases} 0 & \text{if } (x=0) \text{ or } (y=0) \\ f_{\mathcal{R}}(x,y) & \text{otherwise} \end{cases}.$$

*The function $f_{\mathcal{R}}(x,y)$ is defined as follows:*

$$\begin{aligned} & f_{\mathcal{R}}(x,y) = \\ & \quad 6y + \frac{3}{4}(1-2x+2x^2)\left[\frac{y}{y-1} + \frac{y^2}{(x-1)x}\right] + \\ & \quad \frac{3}{2}\left[(2x-1)y(1+y)\log\left(\frac{1-x}{x}\right) + y(1-2x+2x^2+y)\log\left(\frac{1-y}{y}\right)\right]. \end{aligned}$$

We remark that the expression of $f_m(x,y)$ above is valid only for $(x,y) \in \mathcal{R} = \{(x,y) \in [0,1]^2 \mid (x \geq y) \wedge (x \leq 1/2)\}$. The expression of $f_m(x,y)$ on the remainder of $[0,1]^2$ can be

[4] Indeed, the authors consider a generalized version of the random waypoint model, in which nodes are allowed to remain stationary during the entire network operational time with a given probability $p_{stat}$, and the pause time is chosen independently at random at each waypoint. Up to straightforward modifications, our characterization can be extended to this generalized RWP model also.

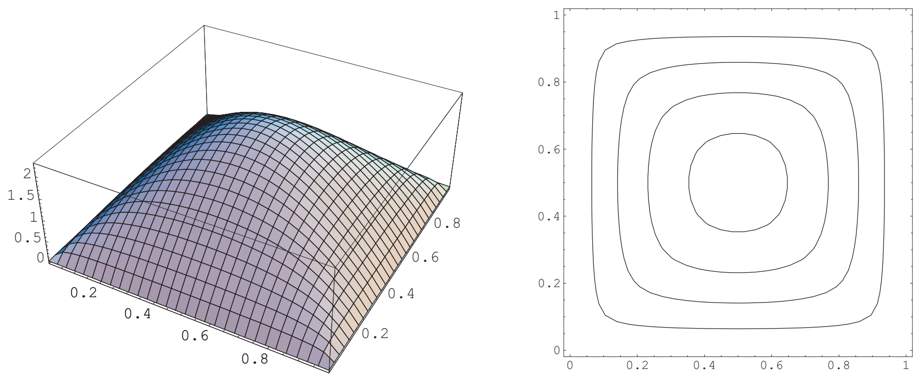


Figure 2: Three-dimensional plot of $f_m(x, y)$, and contour lines corresponding to the values $f_m(x, y) = .5$, 1, 1.5 and 2.

easily obtained observing that by symmetry we have $f_m(x, y) = f_m(y, x) = f_m(1-x, y) = f_m(x, 1-y)$. The 3D and contour plot of $f_m(x, y)$ are reported in Figure 2, which is taken from [3].

The pdf $f_{RW}$ is composed by the sum of two distinct components: the first component accounts for the time a node is resting at the waypoint, and it is uniform because in the RWP model the waypoints are chosen uniformly at random; the second component is not uniform, and is responsible for the border effect.

Note that the non-uniform component of $f_{RW}$ has a minimum value of 0 on the boundary $\partial R$ of $R$. Since the other component of $f_{RW}$ is uniform, it follows that the minimum value of $f_{RW}$ is achieved on $\partial R$. It is immediate to see that the actual value of this minimum equals $P_p = \frac{p}{p+\frac{0.521405}{v}}$. We can then state the following corollary of Theorem 3:

**Corollary 2.** *Let $f_{RW}^p$ denote the long-term node spatial density generated by RWP mobile networks with pause time equal to $p$ and $v_{min} = v_{max} = v$. The minimum value of $f_{RW}^p$ is achieved on $\partial R$, and it equals $P_p = \frac{p}{p+\frac{0.521405}{v}}$. When $p \to \infty$, $f_{RW}^p$ becomes the uniform* *distribution on* $[0,1]^2$*, and* $\min_R f^{\infty}_{RW} = 1$.

We are now ready to prove the following theorem:

**Theorem 4.** *The critical transmitting range for connectivity of an ad hoc network whose nodes move in* $R = [0,1]^2$ *according to the RWP model when the pause time is* $p$ *and* $v_{min} = v_{max} = v$ *is* $r^w_p = \frac{p + \frac{0.521405}{v}}{p}\sqrt{\frac{\ln n}{\pi n}}$ *with high probability if* $p > 0$*. When* $p = 0$*, we have* $r^w_0 \gg \sqrt{\frac{\ln n}{n}}$ *with high probability.*

*Proof.* If $p > 0$, the proof follows immediately by theorems 1 and 2 and Corollary 2. When $p = 0$, the minimum of $f^0_{RW}$ on $R$ is 0 (Corollary 2), and the limit in the statement of Theorem 1 must be interpreted as $+\infty$. □

In words, Theorem 4 states that, as long as the node spatial distribution has a non-null uniform component, the critical transmitting range with RWP mobility differs from the stationary one at most by a constant factor. On the contrary, when the uniform component is null there is an asymptotic gap between the mobile and stationary case.

As observed in [18], the critical range to achieve $k$-connectivity, for any constant $k$, should have the same asymptotic behavior as $M_n$ also in the case of arbitrary node distribution. Hence, a result similar to that of theorems 2 and 4 should hold for $k$-connectivity also.

# 5 Experimental evaluation

In order to investigate whether the asymptotic result of Theorem 4 holds for reasonable values of $n$, we have performed some simulations.

The simulator takes as input the number $n$ of nodes to distribute, the number $\sharp sim$ of simulations to run and the number $\sharp steps$ of mobility steps to perform for each simulation. The simulator also takes as input the parameters of the random waypoint model, i.e. $t_p$ (expressed as the number of steps that the node remains stationary between two movements), and the minimum ($v_{min}$) and maximum ($v_{max}$) velocity (expressed in units of length per step). Initially, nodes are distributed uniformly and independently at random in $[0,1]^2$; then, they start moving according to the random waypoint mobility model. At the end of the $\sharp steps$ steps of mobility, the longest MST edge is calculated and recorded in the output file. The values contained in the output file are used to determine the critical transmitting range, which is defined as the 0.99 quantile of the experimental data. We recall that the $q$ quantile of a series of data gives the location before which $100q$ percent of the data lie.

First, we have investigated the rate of convergence of the CTR to the asymptotic value stated in Theorem 4. We have set $v_{min} = v_{max} = 0.01$, considered two values of the pause time ($p = 100$ and $p = 200$), and computed the asymptotic value of the CTR, which is $1.52\sqrt{\frac{\ln n}{\pi n}}$ when $p = 100$ and $1.26\sqrt{\frac{\ln n}{\pi n}}$ when $p = 200$. Then, we have performed a large number of simulations to evaluate the CTR experimentally: we have distributed a number of nodes ranging from 10 to 2500, and we have performed 10000 mobility steps. As the experimental analysis of [3] has shown, this number of mobility steps ensures the convergence to the asymptotic node spatial distribution. The results of our simulations, averaged over 10000 experiments, are shown in figures 3 and 4. The rate of convergence

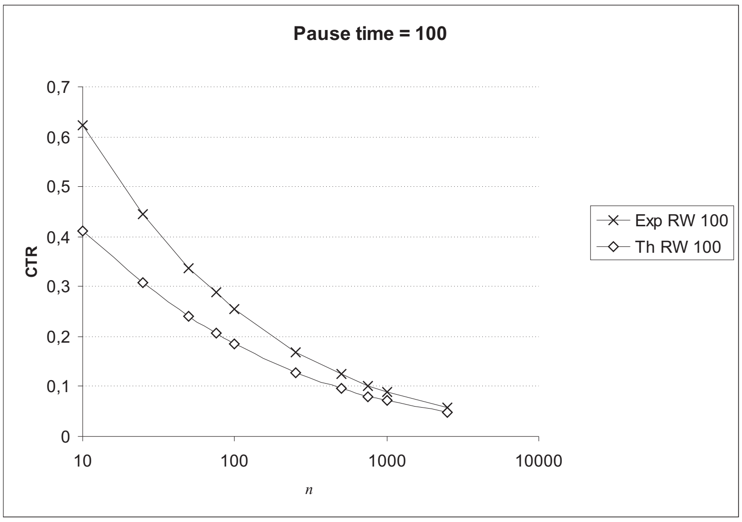


Figure 3: Theoretical and experimental value of the critical transmitting range for values of $n$ ranging from 10 to 2500 when the pause time is set to 100 and $v_{min} = v_{max} = 0.01$. The $x$-axis is in logarithmic scale.

to the asymptotic value is quite slow for both settings of the pause time: only for large networks ($n = 1000$ and above) the value obtained with the formula of Theorem 4 is quite accurate.

We have also experimentally verified the *quality* of Theorem 4 which, we recall, states that the CTR in case of RWP mobility has a different asymptotic behavior with respect to the uniform case only when the pause time is 0. To this purpose, we have simulated four different scenarios. First, we have considered the stationary case, obtained by setting $\sharp steps = 1$. Then, we have set $v_{min} = v_{max} = 0.01$, and considered three settings of $p$: 0, 100, and 200. In the first case, $f_{RW}$ is composed only of the non-uniform component, and the border effect is maximum. In the second case, the intensity of the uniform component

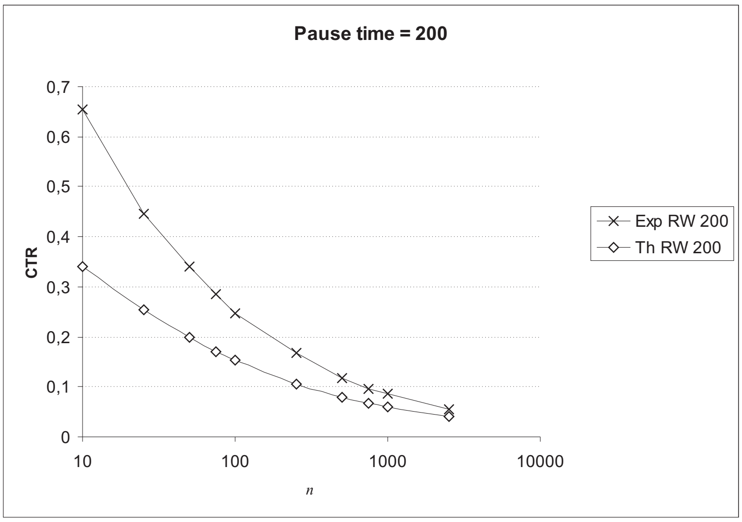


Figure 4: Theoretical and experimental value of the critical transmitting range for values of $n$ ranging from 10 to 2500 when the pause time is set to 200 and $v_{min} = v_{max} = 0.01$. The $x$-axis is in logarithmic scale.

of $f_{RW}$ equals $P_{100} = 0.657$; with $p = 200$, the intensity of the uniform component is $P_{200} = 0.793$. In each scenario considered, we have distributed a number of nodes ranging from 10 to 2500 and, in case of mobility, we have performed 10000 mobility steps.

The results of our simulations, averaged over 10000 experiments, are reported in Table 1, and plotted in Figure 5 and 6. As it is seen, the qualitative result of Theorem 4 is fully confirmed: the curves referring to the stationary scenario and to the case of RWP mobility with positive pause times are very close to each other (see Figure 6), while that corresponding to the extreme case of mobility ($p = 0$) is far apart (see Figure 5). As predicted by Theorem 4, the critical transmitting range in case of "uniform" mobility is larger than in the stationary case, although only marginally. Note that setting $p = 100$

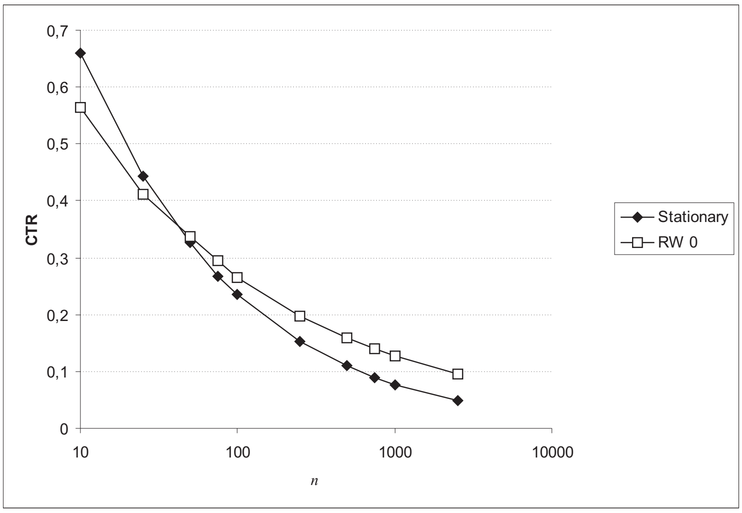


Figure 5: Critical transmitting range in the stationary case and with extreme RWP mobility for values of $n$ ranging from 10 to 2500. The $x$-axis is in logarithmic scale.

is already sufficient to render the CTR very close to the stationary one. Conversely, the critical transmitting range in presence of extreme mobility is much larger than in the stationary case for large values of $n$. Quite interestingly, the situation is reversed for small values of $n$ ($n \leq 50$). This is due to the fact that when $n$ is small, the probability of finding at least one node close to the border is very low, and the critical transmitting range is smaller than in the stationary case (nodes are concentrated in the center of the deployment region). However, when $n$ is large enough, some of the nodes actually lie close to the border of $R$, causing the asymptotic gap with respect to the uniform case.

We have found that, given the value $r$ of the CTR in the stationary case, the critical range in case of extreme mobility is well approximated by the formula $r_0^w = \frac{r}{4} \ln n$, for sufficiently large $n$ ($n \geq 50$). Figure 7 reports the plot obtained with our formula and that produced by the simulations. For $n \geq 100$, our approximation provides a quite accurate

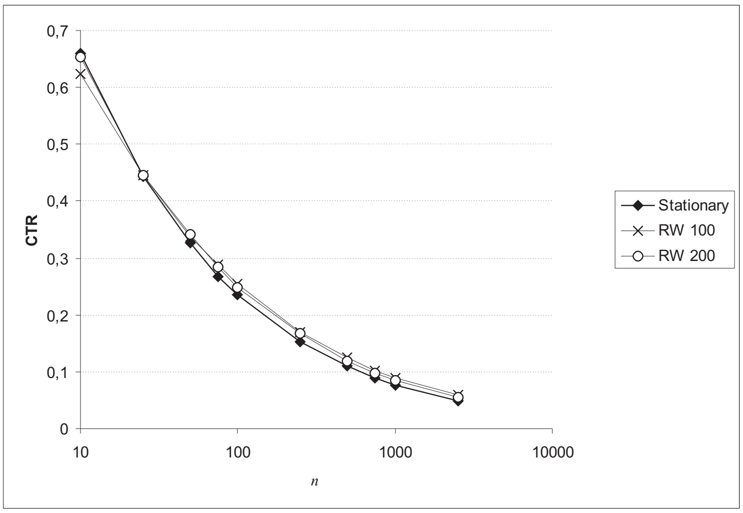


Figure 6: Critical transmitting range in the stationary case and with intermediate RWP mobility ($p = 100$ and $p = 200$) for values of $n$ ranging from 10 to 2500. The $x$-axis is in logarithmic scale.

upper bound to the actual CTR. Table 2 reports the relative error of our formula with respect to the actual CTR, defined as $\frac{r_{app}-r_0^w}{r_0^w}$, where $r_{app}$ is the approximate value of the CTR. We observe that the relative error is below 6.4% when $n \geq 50$.

We remark that the formula above can be used to improve the accuracy of mobile network simulation. As noticed in the Introduction and proved in this paper, setting the nodes' transmitting range to the critical value computed as if the nodes were uniformly distributed is likely to produce inaccurate results in many scenarios (for instance, when simulating an ad hoc routing protocol in presence of RWP mobility). Given the critical range $r$ with uniformly distributed nodes, our formula can be used to compute a tight upper bound on the CTR in presence of RWP mobility, thus improving considerably the accuracy of the simulation results. The fact that our formula depends on $r$ is not a

| n | Uni | RWP 200 | RWP 100 | RWP 0 |
|---|---|---|---|---|
| 10 | 0.6587 | 0.65422 | 0.62409 | 0.56423 |
| 25 | 0.44249 | 0.44531 | 0.44454 | 0.41203 |
| 50 | 0.32763 | 0.34132 | 0.33717 | 0.33644 |
| 75 | 0.268 | 0.28439 | 0.28761 | 0.,29454 |
| 100 | 0.23494 | 0.2478 | 0.25395 | 0.26526 |
| 250 | 0.15183 | 0.16691 | 0.16933 | 0.19761 |
| 500 | 0.10926 | 0.11812 | 0.12442 | 0.15955 |
| 750 | 0.08838 | 0.09676 | 0.10216 | 0.13963 |
| 1000 | 0.07727 | 0.08543 | 0.08978 | 0.12708 |
| 2500 | 0.04977 | 0.05478 | 0.0586 | 0.09482 |

Table 1: Values of the CTR for increasing values of $n$. The columns refer to the case of stationary (uniformly distributed) nodes, and to the case of RWP mobility with different settings of the pause time.

problem, since the value of the CTR with uniformly distributed nodes is a widely studied network parameter (see, for instance, the table reported in [24]).

# 6 Concluding remarks

In this paper, we have investigated the critical transmitting range for connectivity in mobile ad hoc networks. We have proved that in presence of bounded and obstacle free

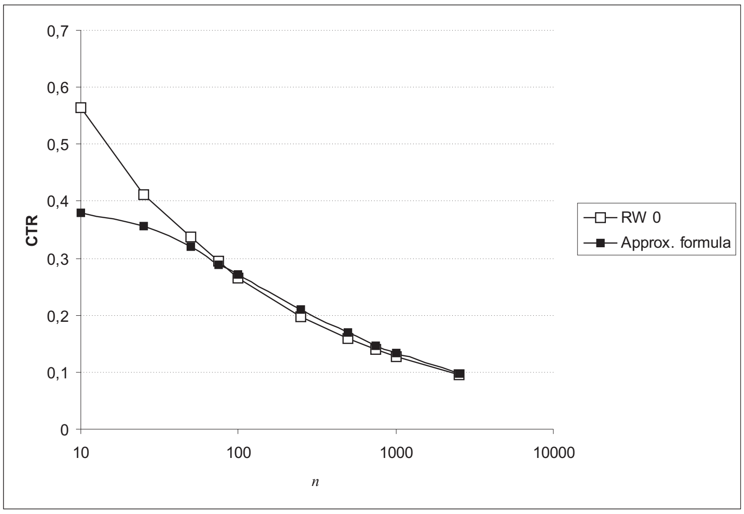


Figure 7: Critical transmitting range with RWP mobility when $p = 0$ (RW 0), and values of the CTR as computed with our formula (Approx. formula). The values of $n$ range from 10 to 2500, and are reported in logarithmic scale.

mobility the CTR in the mobile case is at least as large as the CTR in case of uniformly distributed points (asymptotically). For the case of RWP mobility, we have proved a more accurate characterization of the CTR, and shown that if the pause time is 0 there is an asymptotic gap between the mobile and uniform scenario.

We have verified the quality of our results through simulation. We have also presented a formula that, given the value of the CTR in the uniform case, provides a good approximation of the CTR in the most extreme case of RWP mobility, i.e. when the pause time is set to 0.

We want to remark that the approach presented in this paper can be easily extended to other mobility models: if the expression of the pdf $f_{\mathcal{M}}$ that resembles the long-term node distribution is known and satisfies certain properties, it is sufficient to compute the minimum value of $f_{\mathcal{M}}$ on $R$ to determine the value of the critical range for connectivity.

We believe that the results presented in this paper provide a better understanding of the behavior of a fundamental network parameter in presence of mobility, and in particular of RWP mobility. From a practical point of view, our results can be used to improve the accuracy of RWP mobile ad hoc networks simulation, which is commonly used to evaluate the performance of ad hoc networking protocols.

| n | Relative error |
|---|---|
| 10 | -0.32797 |
| 25 | -0.13579 |
| 50 | -0.0476 |
| 75 | -0.01789 |
| 100 | 0.019696 |
| 250 | 0.060578 |
| 500 | 0.063942 |
| 750 | 0.047558 |
| 1000 | 0.050052 |
| 2500 | 0.026689 |

Table 2: Relative error obtained using our formula.

# 7 Acknowledgement

The author wishes to thank Christian Bettstetter for observing that Penrose's theorem has a wider range of application (besides RWP mobility).

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