The k-NEIGH Protocol for Symmetric Topology Control in Ad Hoc Networks

Abstract

Topology control, wherein nodes adjust their transmitting ranges to conserve energy, is an important feature in wireless ad hoc networks. In this paper, we present a topology control protocol that is fully distributed, asynchronous, and localized. This protocol, referred to as the k-NEIGH protocol, maintains the number of neighbors of every node equal to or slightly below a specific value k. Furthermore, the protocol ensures that the resulting communication graph is symmetric, thereby easing the operation of higher protocol layers. To evaluate the performance of the protocol, the value of k that ensures a connected communication graph with high probability is evaluated. It is also shown that, with n nodes in the network, the protocol terminates on every node after exactly $2n$ messages total and within strictly bounded time. Finally, extensive simulations are carried out, which show that the k-NEIGH protocol is about 20% more energy-efficient than the most widely-studied existing protocol.

1 Introduction

It is a widely accepted fact that the limited energy available at the nodes of a wireless ad hoc network must be used as efficiently as possible. If energy conservation techniques are used at different levels, the functional lifetime of both individual nodes and the network can be extended considerably. For this reason, energy conserving protocols at the MAC, routing, and upper layers have been proposed [6, 11, 14, 28]. Further energy can be saved if the network topology itself is energy-efficient, i.e., if the nodes' transmitting ranges are set in such a way that a target property (e.g., connectivity) of the resulting network topology is guaranteed, while the global energy consumption is reduced. A protocol that attempts to achieve this is called a *topology control* protocol. Several examples of topology control mechanisms have been recently introduced [1, 5, 10, 15, 16, 21, 23, 27].

In order to be easily implementable in a realistic scenario, a topology control protocol should be *fully distributed*, *asynchronous*, and *localized* (i.e., the computation at every node should be based on information concerning neighbor nodes only). These features in general ensure that the protocol is fast and requires little message exchange; thus, it can be easily adapted to deal with dynamic and/or mobile networks. Another desirable property of a topology control protocol is that it does not rely on information that can be provided only by expensive devices, such as location information generated by a GPS receiver. In these conditions, the task of ensuring a global network property (e.g., connectivity) while reducing energy consumption is challenging.

In this paper, we introduce k-NEIGH, a fully distributed, asynchronous, and localized topology control protocol that generates a topology which is proved to be connected with high probability. Contrary to the case of existing protocols, where the number of messages needed to coordinate nodes is not bounded, our protocol exchanges exactly $2n$ messages, where n is the number of nodes. Another positive feature of k-NEIGH is that it is based on distance estimation only, which can be implemented at a reasonable cost in many realistic scenarios. We have performed several simulations, which have shown that our protocol reduces energy consumption considerably with respect to the case where no topology control is used, and that it compares favorably with the CBTC protocol of [15, 27].

2 Related work

In [23], Rodoplu and Meng presented a distributed topology control algorithm that leverages on position information (provided by low-power GPS receivers) to build a topology that is proved to minimize the energy required to communicate with a given master node. Unfortunately, the protocol relies on global knowledge and specialized hardware (the GPS receiver), which makes it infeasible in many application scenarios. Further, the topology generated by the Rodoplu and Meng protocol (which is optimal for communications directed towards a single master node) can be significantly different from the energy optimal topology for the all-to-all communication scheme.

In [27], Wattenhofer et al. introduced a dis-

tributed topology control protocol based on directional information, called CBTC (Cone Based Topology Control). The basic idea is that a node i transmits with the minimum power $p_{i,\rho}$ such that there is at least one neighbor in every cone of angle ρ centered at i. The obtained communication graph is made symmetric by adding the reverse edge to every asymmetric link. The authors show that setting $\rho \leq 2\pi/3$ is a sufficient condition to ensure connectivity. A set of optimizations aimed at pruning energy-inefficient edges without impairing connectivity (and symmetry) is also presented. Further, the authors prove that if $\rho \leq \pi/2$, every node in the final communication graph has degree at most 6. A more detailed analysis of CBTC, along with an improved set of optimizations (which, however, rely on distance estimation), can be found in [15]. The CBTC protocol has been extended to the three-dimensional case in [1]. The authors of [1] also presented a modification of the protocol aimed at ensuring k-connectivity. In [10], the CBTC protocol is implemented using directional antennas.

In [5], Borbash and Jennings introduced a protocol which is also based on directional information. The goal of the protocol is to build the Relative Neighbor Graph of the network in a distributed fashion. The choice of the RNG as the target graph of the protocol is due to the fact that it guarantees connectivity and it shows good performance in terms of average transmitting range, node degree and hop diameter.

The protocols that are most closely related to our work are the MobileGrid protocol of [16] and the LINT protocol of [21]. Both protocols try to keep the number of neighbors of a node within a low and high threshold centered around an optimal value. When the actual number of neighbors is below (above) the threshold, the transmitting range is increased (decreased), until the number of neighbors is in the proper range. However, for both protocols no characterization of the optimal value of the number of neighbors is given, and, consequently, no guarantee on the connectivity of the resulting communication graph is provided. Another problem of the MobileGrid and LINT protocols is that they estimate the number of neighbors by simply overhearing control and data messages at different layers. This approach has the advantage of requiring no overhead, but the accuracy of the resulting neighbor number estimate heavily depends on the traffic present in the network. In the extreme case, a node which remains silent is not detected by any of its actual neighbors.

Similarly to MobileGrid and LINT, the goal of our k-NEIGH is to keep the number of neighbors of a node equal to, or slightly below, a given value k. In contrast with the existing approaches, we formally characterize the optimal value of k, i.e., the minimum value of k such that the communication graph generated is connected with high probability (w.h.p.). We also guarantee that the resulting communication graph is symmetric, i.e., that a node i communicates directly with a node j if and only if j communicates directly with i. Furthermore, we introduce an optimization phase which is inspired by the set of optimizations of [27], and requires no further message exchange between nodes. We prove that our optimization procedure preserves both connectivity and symmetry.

Note that MobileGrid and LINT make no effort to render the communication graph symmetric. As the support of unidirectional links is in general technically difficult and expensive (in terms of number of messages exchanged), we believe that the explicit requirement for a connected backbone of bidirectional links is vital in the design of a topology control mechanism. For further motivations of our symmetry requirement see Section 5.

Compared with CBTC, our k-NEIGH protocol relies on a weaker assumption (distance estimation vs. directional information). Furthermore, CBTC has no bound on the number of messages nor on the energy expended in determining the proper transmit power, whereas in our algorithm each node transmits only two messages at a pre-defined power (the maximum transmit power). Finally, the simulation results reported in Section 6.3 show that the topologies generated by our protocol are (on the average) 20% more energy efficient than those generated by CBTC.

3 Preliminaries

Let N be a set of n nodes placed in $[0,1]^2$ according to some distribution. A *range assignment* for N is a positive real valued function $RA : N \rightarrow (0, r_{max}]$ that assigns to every element of N a value in $(0, r_{max}]$, representing its transmitting range. Parameter r_{max} is called the *maximum transmitting range* of the nodes in the network and depends on the features of the radio transceivers equipping the nodes. We assume that all the nodes are equipped with transceivers having the same features; hence, we have a single value of r_{max} for all the nodes in the network.

Given N and a range assignment RA, the *communication graph* induced by RA on N is defined as the directed graph $G = (N, E)$, where the directed edge $[i, j]$ exists if and only if $RA(i) \geq \delta(i, j)$, and $\delta(i, j)$ denotes the distance between nodes i and j.

In this paper, we are concerned with two variants of this graph, defined as follows:

Definition 1. *The symmetric super-graph of G is defined as the undirected graph G^+ obtained from G by adding the undirected edge (i,j) whenever edge $[i,j]$ or $[j,i]$ is in G. Formally, $G^+ = (N, E^+)$, where $E^+ = \{(i,j)|([i,j] \in E)$ or $([j,i] \in E)\}$.*

Definition 2. *The symmetric sub-graph of G is defined as the undirected graph G^- obtained from G by removing all the non-symmetric edges. Formally, $G^- = (N, E^-)$, where $E^- = \{(i,j)|([i,j] \in E)$ and $([j,i] \in E)\}$.*

The set of neighbors of a node i, denoted $N(i)$, is defined as the set of nodes to which i is directly connected, i.e. $N(i) = \{j|[i,j] \in E\}$. Neighbor sets are defined similarly in graphs G^+ and G^-. Note that for these graphs $i \in N(j)$ if and only if $j \in N(i)$.

Given a parameter k, with $0 < k < n$, the *k-neighbors* graph is the communication graph G_k in which every node is directly connected to its k nearest nodes. Formally, G_k is the communication graph induced by the range assignment RA_k, where $RA_k(i) = \delta(i,j)$ and j is the k-th nearest node to node i.

Several connectivity problems on the communication graph have been studied in the literature. Before formally defining these problems, which are related to some extent to the problem considered in this paper, we need some further definitions.

A range assignment RA is said to be *connecting* if it induces a strongly connected communication graph, while it is said to be *r-homogeneous* if all the nodes have the same transmitting range r, with $0 < r \leq r_{max}$.

It is known [18] that the power p_i required by node i to correctly transmit data to node j must satisfy inequality $\frac{p_i}{\delta_{ij}^{\alpha}} \geq \beta$, where $\alpha \geq 2$ is the *distance-power gradient* and $\beta \geq 1$ is the *transmission quality* parameter. In ideal conditions we have $\alpha = 2$; however, in general it is $2 \leq \alpha \leq 6$ depending on environmental conditions. Setting $\beta = 1$, we can define the *energy cost* of a range assignment RA as $c(RA) = \sum_{i \in N} (RA(i))^{\alpha}$.

We are now ready to formally define the range assignment problems:

Definition 3. *Let N be a set of points in $[0,1]^2$, and assume that the r_{max}-homogeneous range assignment is connecting.*
RA: *Determine a connecting range assignment RA such that $c(RA)$ is minimum.*
WSRA: *Determine a range assignment RA such that the symmetric sub-graph of the communication graph is connected and $c(RA)$ is minimum.*
HRA: *Determine the minimum value of r such that the r-homogeneous range assignment is connecting.*

Note that HRA can be equivalently restated as the problem of finding a connecting homogeneous range assignment of minimum energy cost.

RA and WSRA have been shown to be NP-hard in the two and three-dimensional cases [3, 7, 13], while HRA can be easily solved if node positions are known. HRA has been studied also in the case of nodes distributed according to some probability distribution [9, 24].

In this paper, we are concerned with the following connectivity problem on the symmetric sub-graph of the k-neighbors graph. Motivations for our interest in G_k^- can be found in Sections 4 and 5.

Definition 4 (KNRA). *(Same assumptions as in Definition 3). Determine the minimum value of k such that G_k^- is connected.*

As in the case of HRA, the problem can be equivalently restated in terms of minimum energy cost; furthermore, the optimal solution can be easily found if node positions are known. In the next section, we analyze KNRA in the hypothesis that nodes are distributed uniformly at random in $[0,1]^2$. Our analysis will be used to provide a (probabilistic) guarantee on the connectivity of the topology generated by our k-NEIGH protocol.

4 The minimum number of neighbors for connectivity

A formal analysis of the conditions on the value of k necessary and sufficient to obtain a strongly connected k-neighbors graph (under the hypothesis that nodes are distributed uniformly at random in $[0,1]^2$) is not straightforward. The problem is somewhat simplified if we consider one of the symmetric variants of G_k. In [29], Xue and Kumar proved the following theorem regarding the symmetric super-graph of G_k.

Theorem 1. *Assume that n nodes are placed uniformly at random in $[0,1]^2$, and let G_k^+ be the symmetric super-graph of the k-neighbors graph. There exist two constants c_1, c_2, with $0 < c_1 < c_2$, such that:*

$$\lim_{n \to \infty} Prob\{G^+_{c_1 \log n} \text{ is disconnected}\} = 1 \ , \text{ and}$$

$$\lim_{n \to \infty} Prob\{G^+_{c_2 \log n} \text{ is connected}\} = 1 \ .$$

The authors also provide explicit values for c_1 and c_2, which are $c_1 = 0.074$ and $c_2 > 5.1774$.

Although the difference between the number of neighbors necessary and sufficient for connectivity is quite large, Theorem 1 is very important, since it states that $\Theta(\log n)$ neighbors are necessary and sufficient for connectivity w.h.p. However, Theorem 1 refers to the symmetric super-graph of G_k, in which a link that is physically unidirectional is considered as bidirectional. In other words, the connectivity of G_k^+ is in general higher than that of G_k, since in G_k^+ there are links that do not exist in the actual communication graph. As a consequence, the number of neighbors stated as sufficient to obtain connectivity w.h.p. in Theorem 1 may not be so in the actual communication graph.

In order to circumvent this problem, we consider the symmetric sub-graph G_k^- of G_k, in which all the symmetric edges do exist in the actual communication graph. The following Theorem shows that the same result of Theorem 1 holds also for G_k^-, and thus for G_k.

Theorem 2. *The same result of Theorem 1, with G_k^+ replaced by G_k^-.*

Proof. The necessity part follows immediately by Theorem 1, since $G^-_{c_1 \log n}$ is a sub-graph of $G^+_{c_1 \log n}$. To prove the sufficiency part, we have to show that the construction used in the proof of Theorem 1 holds for $G^-_{c_1 \log n}$ also.
The proof of Theorem 1 is based on the fact (proved in [29]) that any node in $G^+_{c_2 \log n}$ is directly connected w.h.p. to every node that is within distance of $(1-\epsilon)r_n$, where $r_n = \sqrt{\frac{\eta \log n}{\pi n}}$, ϵ is an arbitrary constant in $(0,1)$, and η is a constant that depends on ϵ. In words, this means that the communication graph $G_{(1-\epsilon)r_n}$ generated by the $(1-\epsilon)r_n$-homogeneous range assignment is a sub-graph of $G^+_{c_2 \log n}$ (asymptotically, for $n \to \infty$). Since $G_{(1-\epsilon)r_n}$ is connected w.h.p. (for $n \to \infty$) by Theorem 3.2 in [9], then $G^+_{c_2 \log n}$ is also connected w.h.p.. The proof of our Theorem follows immediately by observing that, since any node is directly connected w.h.p. to every node that is within distance of $(1-\epsilon)r_n$, and distance is obviously symmetric, $G_{(1-\epsilon)r_n}$ is a sub-graph of $G^-_{c_2 \log n}$ too. □

5 The k-NEIGH protocol

In this section, we describe the k-NEIGH topology control protocol, which is based on the following assumptions:
1. nodes are stationary;
2. the maximum transmission power P is the same for all the nodes;
3. given n, P is chosen in such a way that the communication graph that results when all the nodes transmit at power P is connected w.h.p.;
4. a distance estimation mechanism, possibly error prone, is available to every node;
5. the nodes initiate the k-NEIGH protocol at different times. However, the difference between node wake up times is upper bounded by a known constant Δ.

Assumption 4 is the most critical in our model and deserves some comment. The distance estimation techniques proposed in the literature so far are based on:
– *Radio Signal Strength*: distance is estimated comparing the transmitted power at the sender (which is piggybacked in the message) and the received power at the receiver of the message. This technique can be implemented at virtually no cost (RSSI registers are a standard feature in many wireless network cards [25]), but provides poor accuracy. In [25], it is shown that RSSI-based distance estimation is feasible only in a quite idealized setting (football field with all the nodes positioned at the ground level).
– *Time of Arrival*: distance is estimated comparing the time of arrival of different kinds of signals. Typically, the radio signal is used in combination with acoustic, ultrasound or infrared signals. ToA-based techniques provide a much better accuracy than RSSI-based mechanisms, and can be implemented at a reasonable cost. For example, the technique proposed in [8] uses a standard PC sound card to generate an acoustic signal, which is received by a cheap microphone. The authors show that this technique provides good accuracy (below 3%) in realistic conditions. However, accuracy drops to only 23% when the line of sight between the nodes is obstructed by heavy obstacles. In order to overcome this problem, several signals of different kind can be combined together.

Given the discussion above, we can state that Assumption 4 is technically and economically realistic in many scenarios. On the other hand, the impact of errors in distance estimation cannot be disregarded. For this reason, we have included realistic distance estimation error models in our simulator (see Section 6.2 for details).

We remark that some of the topology control protocols introduced so far are based on assumptions even stronger than Assumption 4. For example, the protocol of [23] is based on location information, which is provided by a GPS receiver. Although their cost has decreased in recent years, and their form factor reduced, GPS receivers are still ex-

pensive and cumbersome devices. Furthermore, the GPS signal can be received only in open air environments. The protocol of [27] and all of its variants [1, 10, 15], and the protocol of [5], are based on directional information, which can be provided using directional antennas (which are also very expensive).

The goal of our k-NEIGH protocol is to set nodes' transmitting ranges based on local information only, in such a way that the resulting symmetric subgraph G_k^- is connected w.h.p. The choice of limiting our consideration to G_k^- is motivated by the following reasons:

– although implementing wireless unidirectional links is technically feasible (see [2, 12, 19, 20, 22] for unidirectional link support at different layers), the actual advantage of using unidirectional links is questionable. For example, in [17] Marina and Das have shown that the high overhead needed to handle unidirectional links in routing protocols outweighs the benefits that they provide, and better performance can be achieved by simply avoiding unidirectional links;

– a recent theoretical result [3] has shown that the optimal solution to RA and WSRA have the same energy cost (asymptotically). In other words, starting from a strongly connected graph, obtaining a connected backbone of symmetric edges incurs in no additional (asymptotic) energy cost.

Thus, having a connected backbone of symmetric edges, as the k-NEIGH protocol provides, allows us to use standard bidirectional link-based protocols in the upper layers, avoiding the expensive and technically difficult implementation of unidirectional links. Given the theoretical result of [3] and Theorem 2, this additional requirement on the communication graph will come with a limited additional energy cost[1].

In the protocol specification below, we assume without loss of generality that the first node wakes up at time 0. The protocol is as follows:

THE k-NEIGH PROTOCOL (FOR A GENERIC NODE i):

1. Node i wakes up at time t_i, with $t_i \in [0, \Delta]$. At random time t_i^1 chosen in the interval $[t_i + \Delta, t_i + \Delta + d]$ (the value of the parameter d is set in Lemma 1), node i announces its ID at maximum power;
2. For every message received from other nodes, i stores the identity and the estimated distance of the sender;
3. At time $t_i + 2\Delta + d$, i orders the list of its neighbors (i.e., of the nodes from which it has received the announcement message) based on the estimated distance; let L_i be the list of the k nearest neighbors of node i (if i has less than k neighbors, L_i is the list of all its neighbors).
4. At random time t_i^2 chosen in the interval $[t_i + 2\Delta + d + \tau, t_i + 2\Delta + 2d + \tau]$ (τ is an upper bound on the duration of step 3), node i announces its ID and the list L_i at maximum power.
5. At time $t_i + 3\Delta + 2d + \tau$ node i, based on the lists L_j received from its neighbors, calculates the set of *symmetric neighbors*[2] in L_i. Let L_i^S be the list of symmetric neighbors of node i, and let j be the farthest node in L_i^S.
6. Node i sets its transmitting power P_i to the power needed to transmit at distance δ_{ij}^e, where δ_{ij}^e is the estimated distance between nodes i and j.
7. (OPTIONAL PRUNING STAGE) Apply an optimization procedure to reduce the number of edges in the graph obtained so far (see below).

At the end of the protocol execution, node i considers as neighbors (e.g., for the purpose of routing) only the nodes in the list L_i^S. Note that these are *logical* neighbors, and the set of *physical* neighbors in general is larger than L_i^S: when i transmits at power P_i, it is possible that some node $j \notin L_i^S$ receives the message. However, these are asymmetric neighbors, which are not considered. Also, the following pruning stage can be executed to further reduce the number of logical neighbors and (possibly) the actual transmission power required at some node.

PRUNING STAGE OF THE k-NEIGH PROTOCOL (FOR A GENERIC NODE i):

Let $G_k^- = (N, E)$ be the undirected graph obtained as the results of steps 1–6 of the k-NEIGH protocol, and, for any $(i, j) \in E$, let $P(i, j)$ denote the transmission power sufficient for i to reach node j. This information is included in the message sent by node i during step 4.

1. Node i sorts the list L_i^S according to increasing values of $P(i, j)$ (initially, this is equivalent to the order given by the increasing distances from i). Let $j_1, \ldots, j_k$ be the sorted list (without loss of generality, we assume that L_i^S contains k elements; otherwise, the sorted list will be composed by

[1] In Section 6.1 we will validate this statement through extensive simulation.

[2] Nodes i and j are said to be *symmetric neighbors* if and only if $i \in L_j$ and $j \in L_i$.

$k_1 < k$ elements).
2. For $l = 2, \ldots, k$, do the following.

a. Check whether j_l can be reached using a transmission power lower than $P(i, j_l)$ by routing through some j_q, $q < l$. Clearly, given the information available to node i, this is possible only if $(j_q, j_l) \in E$, a circumstance that is known to i from step 5 of the k-NEIGH protocol.

b. If $P(i, j_q) + P(j_q, j_l) \leq P(i, j_l)$, logically delete the (outgoing) edge (i, j_l) and set $P(i, j_l) = P(i, j_q) + P(j_q, j_l)$. If more than one node satisfies this requirement, choose the node q such that $P(i, j_q) + P(j_q, j_l)$ is minimum.

3. Set the transmitting power to the power needed to reach the farthest node in L_i^S which is still an immediate neighbor of node i.

The following results prove that the k-NEIGH protocol is correct.

Lemma 1. *Let $\bar{t}$ be the time necessary to transmit a message. For $d = m\bar{t}$, the probability that no contention will occur in the wireless channel during step 1 of the k-NEIGH protocol is strictly greater than $e^{-\frac{3h(h-1)}{2m}}$, where h is the number of nodes that are contending for the channel when transmission is done at maximum power.*

Proof. In the worst case, all the nodes wake up at the same time $\bar{\Delta} \in [0, \Delta]$, and all the transmissions in step 1 will occur at a time taken uniformly at random in the interval $[\Delta + \bar{\Delta}, \Delta + \bar{\Delta} + d]$. Fix $d = m\bar{t}$, so that the interval $[\Delta + \bar{\Delta}, \Delta + \bar{\Delta} + d]$ can be divided into m sub-intervals of length $\bar{t}$ each. If node i initiates the transmission during the z-th interval (i.e., at some time in $((z-1)\bar{t} + \Delta + \bar{\Delta}, z\bar{t} + \Delta + \bar{\Delta}]$, for some integer $z \in 1 \ldots m$), we say that the z-th interval is occupied. Now, the following is clearly a sufficient condition for the occurrence of the "no contention" event: no pair of nodes occupies the same interval z and, if an interval z is taken, then intervals $z-1$ and $z+1$ are free. Since the transmission times are independent events, we may assume that the "choices" of the transmission intervals made by nodes form a sequence of independent random variables Z_i uniformly distributed in $[1, m]$, with $i = 1, \ldots, h$. A success in the i-th trial occurs when $|Z_i - Z_j| > 1$, for any $j < i$. It is easy to see that this happens with probability at least $\frac{m-3(i-1)}{m}$. The probability of no contention is then lower bounded by

$$Pr\{\text{no contention}\} \geq 1 \cdot \left(1 - \frac{3}{m}\right) \cdot \ldots \cdot \left(1 - \frac{3(h-1)}{m}\right)$$

Taking the logarithms and using the first term of the Taylor expansion of $\log(1 - x)$ at $x = 0$, we have:

$$\log Pr\{\text{no contention}\} \geq \sum_{i=2}^{h} \log\left(1 - \frac{3(i-1)}{m}\right) >$$

$$> -\sum_{i=2}^{h} \frac{3(i-1)}{m} = -\frac{3}{m}\sum_{i=1}^{h-1} i = -\frac{3h(h-1)}{2m}.$$

The proof follows by exponentiation. □

Lemma 1 can be used to lower bound the probability of no contention when accessing the wireless channel. For example, if $n = 100$ nodes are distributed uniformly at random in a square region and P is chosen in accordance with Assumption 3, the expected number of nodes within the maximum transmitting range is about 33 (see Section 6 for details). Given these settings, d must be around $16000\bar{t}$ to obtain a probabilistic guarantee of no contention of at least 0.9. With $\bar{t}$ in the order of, say, milliseconds, d will be in the order of tenth of seconds, which is reasonable for most topology control scenarios. Clearly, Lemma 1 provides only a crude lower bound on $Pr\{\text{no contention}\}$, and smaller values of d can be used in practice.

Let $G = (N, E)$ be a graph. If $i, j \in N$ and there is a path p connecting i to j we write $i \rightsquigarrow_p j$, or simply $i \rightsquigarrow j$, if p is understood.

Lemma 2. *Let $G_k^- = (N, E)$ be the undirected graph computed by steps 1-6 of the k-NEIGH protocol, and suppose G_k^- is connected. Let $G' = (N, E')$ be the directed graph obtained as the result of the pruning stage of k-NEIGH. Then, G' is strongly connected and symmetric.*

Proof. We first prove that G' is strongly connected by showing that, if $(i, j) \in E$ and (i, j) is deleted, then $i \rightsquigarrow j$ will still hold in G'. Consider the pruning stage executed by node i. According to the protocol, node i deletes (i, j) provided that there is a neighbor i_1 of i such that $i \rightsquigarrow_p i_1$, for some path p in G_k^-, and moreover $(i_1, j) \in E$. Now, let p' denote the whole path from i to j (i.e., p' is p plus the final edge (i_1, j)). By the same argument above, if some edge (s, t) of p' is removed in G' as the result of the pruning stage executed by node s, then an alternative path p'' exists in G' that connects s to t, and hence i to j. It is an easy consequence of the cost rule $2b$ of the pruning protocol that all these paths must be acyclic (for otherwise a contradiction would occur by summing the transmission powers on a circuit). Since the number of nodes is finite,

the process of replacing an edge with a path must eventually stop.
As for the symmetry, it is sufficient to observe that if node i deletes (i,j), then $P(i,z)+P(z,j) \leq P(i,j)$, for some node z. The symmetry of the power function P implies that node j will delete (j,i) as well. □

Theorem 3. *Assume that k is chosen in accordance with Theorem 2. Then the k-NEIGH protocol:*
a. terminates at time at most $4\Delta + 2d + \tau$ (where d is set in Lemma 1), i.e., by this time all the nodes have set their transmitting power correctly and terminated the protocol execution;
b. generates a symmetric communication graph which is connected w.h.p. under the hypothesis that nodes are distributed uniformly at random in $[0,1]^2$;
c. has communication complexity $\Theta(n)$.

Proof. A generic node i wakes up at an arbitrary time t_i in $[0,\Delta]$. Before announcing its ID, node i has to wait at least time Δ to avoid that its message is not received by nodes that are not yet awake. The additional random time (in the interval $[0,d]$) is needed to avoid (with high probability) contention in accessing the wireless channel. Once the node has announced its ID at step 1, it has to wait for messages coming from other nodes. The waiting time is $\Delta + d$, accounting for the difference in the initial wake up times and for the maximum possible difference between random time choices. Thus, at time $t_i + 2\Delta + d$, node i can safely order neighboring nodes based on the distance estimated when the announcement messages are received. We recall that messages are sent at maximum power, which is the same for all the nodes by assumption. This implies that at time $t_i + 2\Delta + d$ node i has received the announcement messages of all the nodes within its maximum transmitting range. The ordering phase lasts at most time τ, and at time $t_i + 2\Delta + d + \tau$ node i is ready to send the message containing its k-neighbors list. Once more, the node waits for an additional random time chosen in the interval $[0,d]$ to avoid contention. Before ending the protocol, node i must be sure to have received the k-neighbors lists of all its neighbors, so that asymmetric neighbors can be removed. Thus, starting at time $t_i + 2\Delta + d + \tau$ node i waits for further $\Delta + d$ units of time. At time $t_i + 3\Delta + 2d + \tau$, node i is then ready to set its transmitting power correctly and the protocol execution in node i terminates. The proof of the part *a.* of the theorem follows by observing that the maximum possible value for the wake up time t_i is Δ.
The proof of the part *b.* of the theorem follows by Theorem 2. The part *c.* of the Theorem is immediate, since every node sends exactly two messages. By definition of the pruning stage and by Lemma 2, it is immediate that pruning occurs with no further message exchange, and produces a graph which is connected w.h.p. and symmetric. □

6 Simulation results

To evaluate our k-NEIGH protocol we have designed an ad hoc simulator and performed a considerable body of experiments. The goals of our simulations include evaluation of:
– *preferred value of k*: the result stated in Theorem 2 is mainly of theoretical interest. In the first set of experiments, we have evaluated which values of k should be used in practice to achieve a target probability (e.g., 0.95) of connectivity. We call this value the *preferred value* of k and on the correctness of the protocol;
– *effect of errors*: as discussed in Section 5, distance estimation techniques are error-prone. We have evaluated the effect of errors in distance estimation on the preferred value of k;
– *energy cost*: in the third set of experiments, we have compared the performance of our algorithm (in terms of energy cost, as defined in Section 3) to that of other algorithms.

The results of each set of simulations are presented in separate subsections.

6.1 Preferred value of k

The preferred value of k is defined as the minimum value of the node degree k which guarantees that $Pr(G_k^-$ is connected) is above a certain target probability. Since a theoretical characterization of this value is very difficult, we have evaluated it through extensive simulations.

The setting used for our experiments is the following. The n nodes, all with the same maximum transmitting range R_n, are distributed uniformly at random in $[0,1]^2$. According to Assumption 3 of Section 5, R_n should be chosen so that the communication graph that results when all nodes transmit at maximum power is connected with high probability. An easy choice would be to make R_n independent of n, and sufficiently high to ensure connectivity w.h.p. even with very few nodes (e.g., $R_n = 1$). However, the choice of R_n has a strong influence on the energy cost of the graph generated by topology control algorithms in general. For example, in the CBTC protocol of [27], boundary nodes are very likely to transmit at full power (after the first phase of the protocol). As a consequence, larger values of R_n produce higher energy costs for CBTC.

For this reason, we have decided to choose R_n according to the following procedure: for every value

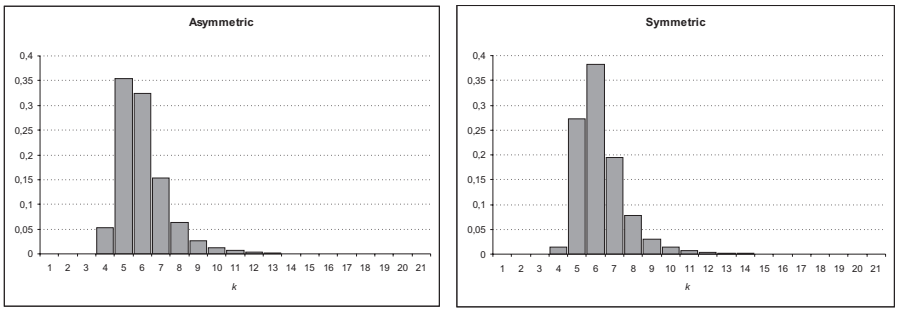

Figure 1: Empirical distribution of the minimum k for connectivity in the asymmetric (left) and symmetric (right) case for $n=100$. Data are shown as frequencies.

n	R_n	n	R_n
10	0.86622	75	0.37041
20	0.66420	80	0.36291
25	0.60431	90	0.34787
30	0.55589	100	0.33326
40	0.48635	250	0.23634
50	0.44526	500	0.19691
60	0.41456	750	0.17885
70	0.38336	1000	0.17274

Table 1: Values of the maximum transmitting range R_n used in our simulations.

of n considered in the simulations, we have generated 10000 random placements and, for every placement, we have evaluated the longest edge of the Euclidean MST.[3] Using these values, we have built the empirical distribution of the critical transmitting range, and taken the 0.99 quantile.[4] This value, further increased by 50% for safety, gives R_n. For all practical purposes, the transmitting range R_n calculated in this way accomplishes Assumption 3, and gives a uniform parameter that can be used in the implementation of k-NEIGH and other topology control protocols. The values of n used in our simulations and the corresponding values of R_n are shown in Table 1.

We have investigated the preferred value of k for different values of n. In the first experiment, n ranged from 10 to 100 in steps of 10. The reason for the small steps of n is that in most ad hoc network applications the number of nodes is expected to be in this range. For every value of n, and for every random node placement, we have calculated the minimum value of k such that G_k is strongly connected (denoted k_{asym}), and the minimum value of k such that G_k^- is connected (denoted k_{sym}), subject to the constraint that every node has maximum transmitting range R_n. Given our choice for R_n, such minimum values for k always exist in practice. For each setting of n, we generated 100000 random node placements, and recorded k_{asym} and k_{sym} for each of them. These data gave us the empirical distribution of k_{asym} and k_{sym}, which can be used to evaluate the preferred value of k. The two distributions for the case of $n=100$ are shown in Figure 1. From the figure, it is evident that the requirement for symmetry has little influence on the minimum value of k for connectivity. This is made clearer by Figure 2, which reports the preferred value of k in the asymmetric and symmetric cases when the target probability of connectivity is set to 0.95. These values can be easily obtained by the cumulative distribution of k_{asym} and k_{sym}: the preferred value is the minimum value of k such that the cumulative frequency is above 0.95.

The plots reported in Figure 2 show that the preferred value of k in the symmetric case is at most 1 greater than the value in the asymmetric case. To a certain extent, this confirms the theoretical results of Theorem 2 and of [3]. Figure 2 also reports the average node degree in the symmetric case. We recall that k is the number of asymmetric neighbors, while only symmetric neighbors contribute to the node degree of G_k^-. The plot seems to confirm the logarithmic behavior predicted by Theorem 2.

In the second experiment, we have evaluated how the preferred value of k varies for larger values of n. We have used the following settings for n: 10, 25, 50, 75, 100, 250, 500, 750, 1000. For every value of n, we have calculated the preferred value of k in the asymmetric and symmetric cases (with target probability 0.95), proceeding as in the previous experiment. The results of this experiment are shown

[3] It is known that this value corresponds to the critical transmitting range, in case the range assignment is homogeneous (see [4]).

[4] We recall that the q quantile of a series of data gives the point such that $100q$ percent of the data lie before.

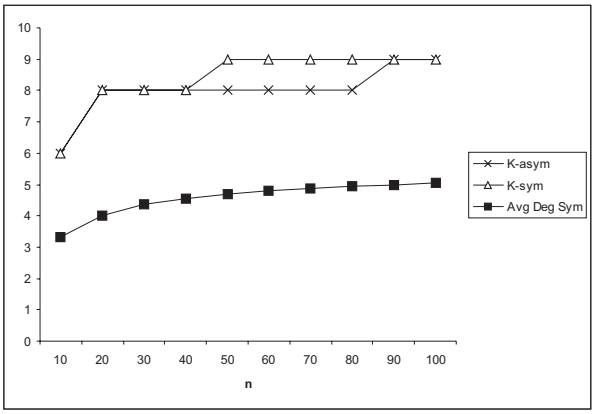

Figure 2: Preferred values of k in the asymmetric and symmetric cases (y-axis), with target probability 0.95, for different values of n (x-axis). The graphic also reports the average node degree in the symmetric case.

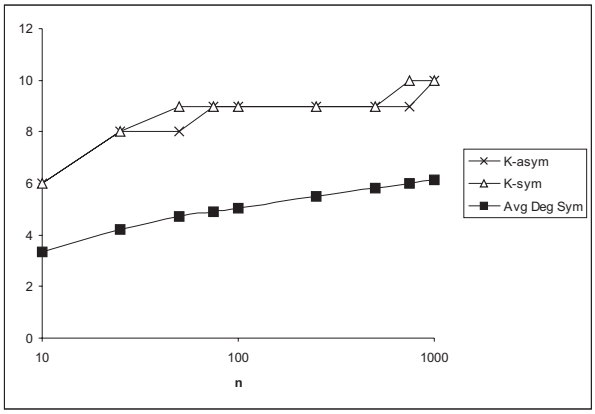

Figure 3: Preferred values of k in the asymmetric and symmetric cases (y-axis), with target probability 0.95, for different values of n (x-axis). The graphic also reports the average node degree in the symmetric case. Values on the x-axis are reported in logarithmic scale.

in Figure 3, along with the average node degree in the symmetric case. Again, the difference between the preferred value of k in the asymmetric and symmetric cases is at most 1, and the two values are the same for many settings of n. Concerning the average node degree in the symmetric case, the logarithmic scaling with n is confirmed.

Interestingly, setting $k=9$ produces a symmetric graph which is connected with probability at least 0.95 for values of n in the range 50–500. In [16], it is shown that when all the nodes have the same transmitting range, a number of neighbors in the range 3–9 is optimal from the network capacity point of view, and it is also close to the optimal value for power efficiency. In this respect, our result can be seen as an improvement of [16], since we achieve connectivity with adaptive transmitting ranges.

A final investigation concerned the number of asymmetric neighbors when $k = k_{sym}$, i.e., in the minimal scenario for achieving connectivity in G_k^-. We recall that asymmetric neighbors (and the corresponding asymmetric links) will be removed by the k-NEIGH protocol. The percentage of asymmetric links removed (over the total number of links) for values of n in the range 10–1000 is reported in Figure 4, which shows a decreasing behavior with n: 22.5% for $n=10$, down to 16.8% for $n=1000$. From our experiment, we observed that the average number of asymmetric links removed per node is slightly above 1.2, independently of n.

Overall, the results of this first set of simulations have shown that the requirement for symmetry has little influence on the preferred value of k, and that setting $k=9$ in the k-NEIGH protocol provides connectivity w.h.p. for a wide range of network sizes (from 50 to 500 nodes).

6.2 Errors in distance estimation

In this section, we investigate how the preferred value of k is influenced by errors in distance estimation. To this purpose, we have implemented two models, which account for errors in RSSI- and ToA-based techniques.

In case of RSSI, error is due to the fact that the propagation of the radio signal in the air is influenced by many factors (weather changes, obstacles, and so on), and, consequently, an accurate model of the signal attenuation with distance is very difficult to obtain. Thus, the transformation of the difference between the transmitted and received power into a distance estimation induces a considerable error, which can be unacceptable in many situations. In [25], it is shown that the accuracy of RSSI-based distance estimation is reasonable only in quite idealized settings, such as all the nodes placed in a flat open environment.

We have modeled the error in RSSI-based distance estimation using the scheme proposed in [26], which is defined as follows:

$$RSSI(\delta) = \delta(1 - 10^{\frac{X_\sigma}{10\alpha}}) \ ,$$

where δ is the actual distance, α is the distance-power gradient, and X_σ is a random variable with normal distribution of parameters $(0, \sigma)$. According to the measurements reported in [25], in our

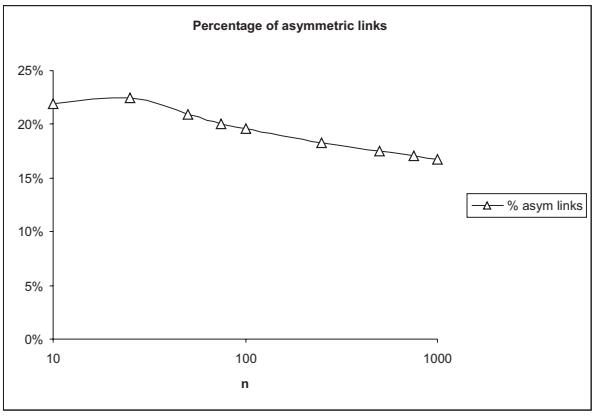

Figure 4: Percentage of asymmetric links removed (y-axis) for different values of n (x-axis). Values on the x-axis are reported in logarithmic scale.

simulations we set $\sigma = 0.84$ and $\alpha = 2$. With these settings, 70% of the estimations are within 10% of the actual distance δ.

To model errors in ToA-based distance estimation, we have simplified the scheme of [26], which is based on the acoustic ranging technique of [8]. In this case, the error can be seen as the sum of three independent components:
– *speed of sound error*: changes in the atmospheric conditions can generate both a positive and a negative error in the distance reading. We denote this error with SSE.
– *Non-Line-Of-Sight error*: this error, which is always positive, occurs when obstacles obstruct the line of sight between nodes. We denote this error with $NLOS$.
– *orientation error*: this error, which is always positive, occurs when the emitter and the receiver of the acoustic signal have different orientations. We denote this error with OE.

In our simulations, we have used the following settings for SSE, $NLOS$ and OE, which are based on the measurements reported in [8]:
– SSE is modeled as a uniform error centered at δ. More precisely, $SSE(\delta) = U[-0.005\delta, 0.005\delta]$, where $U[-0.005\delta, 0.005\delta]$ is a random variable with uniform distribution in the interval $[-0.005\delta, 0.005\delta]$.
– the experiments reported in [8] have shown that, while "light" obstacles (e.g., a stack of small cardboard boxes) have little influence on the accuracy of distance estimation, "heavy" obstacles (e.g., a large mattress) cause a relevant error. In our model, we have considered three types of obstructions: no obstruction, light obstruction, and heavy obstruction. In case of no obstruction, $NLOS(\delta) = 0$; with light obstacles, we have $NLOS(\delta) = U[0.006\delta, 0.01\delta]$, and with heavy obstacles we set $NLOS(\delta) = U[0.18\delta, 0.22\delta]$. For every pair of nodes within each other maximum transmitting range, we perform an independent random experiment, and choose "no obstruction" with probability p_1, "light obstruction" with probability p_2, and "heavy obstruction" with probability $1 - (p_1 + p_2)$. In our experiments, we have set p_1 and p_2 to the values 0.5 and 0.25 respectively, which describe an open air environment with relatively few heavy obstacles. Admittedly, modeling actual NLOS errors (that *are not* independent) is a complicated task, and more investigations are needed on this subject.
– for every pair of nodes within maximum transmitting range, we perform an independent random experiment with four possible equiprobable outcomes, namely 0, 90, 180, 270. These values correspond to an orientation error of 0 degrees, 90 degrees, and so on. We set $OE(\delta) = 0$ when the outcome is 0, $OE(\delta) = U[0.004\delta, 0.006\delta]$ when the outcome is 90 or 270, and $OE(\delta) = U[0.014\delta, 0.016\delta]$ when the outcome is 180. As in the case of NLOS error, our independence assumption introduces a slight approximation, but simplifies the model considerably.

In summary, the ToA-based distance estimation error is defined as follows:

$$ToA(\delta) = SSE(\delta) + NLOS(\delta) + OE(\delta) \ .$$

We have incorporated the two distance estimation error models in the simulator, and performed a set of experiments to evaluate the impact of errors on the preferred value of k. To account for possible errors in distance estimation, the simulator has been modified as follows. For every node, we store two neighbor lists: the list L with the actual distances, and the list L_e with the estimated distances. Both lists are ordered for increasing values of distance. The estimated distances are generated during a preprocessing phase in which, for every pair of nodes within maximum transmitting range R_n, we calculate the estimated distance according to the chosen error model. We assume that errors in distance estimation are *symmetric*: if node i estimates that node j is at distance δ^e_{ij}, also node j performs the same estimation δ^e_{ij}. Since in the k-NEIGH protocol nodes estimate distances to their neighbors in a very narrow time interval, this assumption is coherent with our error models.

Based on the list L_e, node i sets its transmitting power to the value needed to reach the k-th node in the list, say node j, which is at estimated distance δ^e_{ij}. Since δ^e_{ij} is only an estimate of the actual distance, there could exist one or more nodes h such that h precedes j in the list L_e, but $\delta_{ih} > \delta^e_{ij}$. Simi-

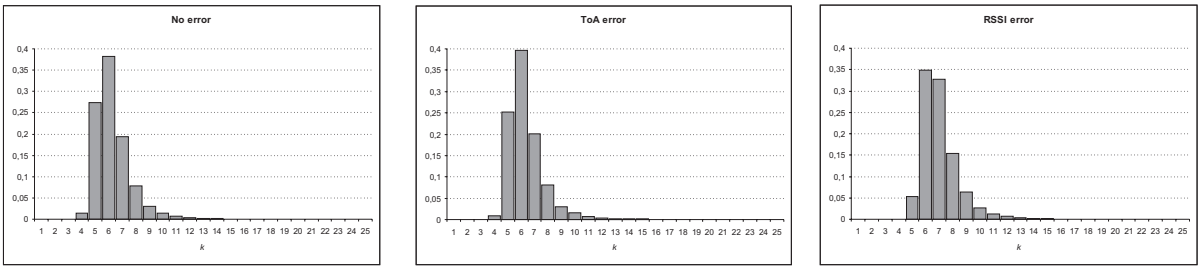

Figure 5: Empirical distribution of the minimum k for connectivity in the exact (left), ToA error (center), and RSSI error (right) case for $n=100$. Data are shown as frequencies.

n	ToA	RSSI	n	ToA	RSSI
10	6	7	10	6	7
20	8	9	25	8	9
30	8	9	50	9	9
40	8	9	75	9	9
50	9	9	100	9	10
60	9	9	250	9	10
70	9	9	500	9	10
80	9	9	750	10	10
90	9	10	1000	10	11
100	9	10			

Table 2: Preferred values of k (with target probability 0.95) with ToA and RSSI error. The values of k with ToA error always coincide with those of the exact case.

larly, there could exist some node v that follows j in L_e, but such that $\delta_{iv} \leq \delta^e_{ij}$. In words, the k-th node in L_e may not be the actual k-th nearest neighbor of i. For this reason, once we have set the transmitting range to δ^e_{ij}, for every node h that precedes j in L_e we check (using the list L of the actual distances) whether the link to h actually exists. Note that, since the k-NEIGH protocol will only consider the first k nodes in L_e, possible links to nodes that follows j in L_e are not included in the generated graph. Once we have generated the (asymmetric) communication graph according to the procedure described above, we consider only symmetric links and check for connectivity, as in the previous set of experiments.

As in Section 6.1, we have simulated networks of sizes in the range 10–100 in steps of 10, and in the range 10–1000, and evaluated the preferred value of k (with target probability 0.95) in case of RSSI and ToA errors. The results of our simulations are reported in Table 2. As expected, ToA distance estimation performs much better than the simpler RSSI technique: for all the values of n considered, the preferred value of k with ToA error was always the same as in the exact case (see Figures 2 and 3). With RSSI error, the preferred value of k is at most 1 greater than the value in the exact scenario, and it is the same value for many settings of n. The relatively little influence of error in distance estimation on the minimum value of k for connectivity is further evidenced in Figure 5, which shows the empirical distribution of k in the exact, ToA error, and RSSI error cases, for $n=100$. The better performance of ToA with respect to RSSI distance estimation is due to the fact that ToA error, although occasionally large when heavy obstacles obstruct the line of sight, is essentially on the positive side. Thus, the situation described above in which a "close" neighbor cannot be actually reached is less likely to occur.

Overall, the results of this second set of experiments have shown that the k-NEIGH protocol is resilient to errors in distance estimation, also in the scenario in which obstacles obstruct the line of sight of a considerable fraction of node pairs.

6.3 Energy cost

In the last set of experiments, we have compared the performance of k-NEIGH with that of other topology control algorithms. The performance is measured in terms of energy cost, which, we recall, is defined as $c(RA) = \sum_{i \in N} (RA(i))^\alpha$, where RA is the range assignment as defined at the end of the protocol execution. The energy cost gives a measure of the "energy efficiency" of the topology generated by a topology control algorithm.

Another important parameter used in the literature to evaluate the protocols is the average node degree. We recall that, besides reducing energy consumption, topology control mechanisms have the positive effect of increasing spatial reuse, which means that fewer nodes are expected to experience contention in accessing the wireless link. Hence, a reduced average node degree in general implies that contention is reduced as well. However, it is important to note that what really matters is the *physical*,

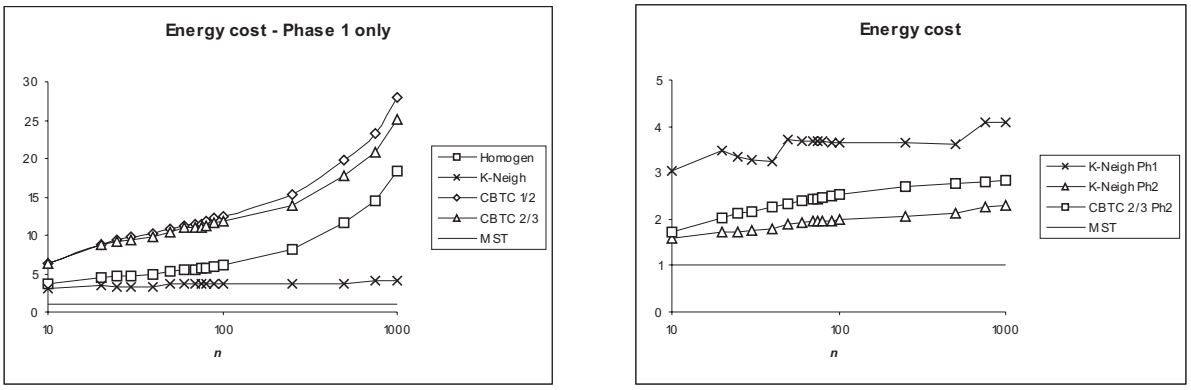

Figure 6: Energy cost of different topology control protocols. For k-NEIGH and CBTC, we have considered Phase 1 only (left), and Phases 1 and 2 implemented (right). The energy cost is normalized with respect to the cost of the MST. Values on the x-axis are reported in logarithmic scale.

rather than the *logical*, degree. In fact, many protocols (such as k-NEIGH and CBTC, for instance) generate a logical topology, in which some of the actual links are not considered, because they are either asymmetric or energy inefficient. Thus, the number of logical neighbors, which determines the logical node degree, could be significantly smaller than the actual number of neighbors, which "measures" the likelihood of contention. Given the same physical degree, a higher logical degree has a positive effect on network capacity, since fewer bottlenecks are likely to occur in the topology. This point has often been disregarded in the previous analyses of topology control protocols, and will be carefully investigated in our simulations.

In our simulations, we have considered values of n ranging from 10 to 1000, as in Sections 6.1 and 6.2. For each value of n, we have generated 10000 random node placements, and executed the following topology control algorithms:

– *MST*: although impractical (its computation requires global knowledge), the Euclidean Minimum Spanning Tree produces a range assignment that is within a factor of 2 from the optimal weakly symmetric range assignment (see [3]). We have used the MST as the "optimal" topology against which the topologies generated by the other protocols will be compared.

– k-NEIGH: for each setting of n, the value of k used in the protocol is the preferred value as evaluated in Section 6.1.

– *CBTC*: we have simulated CBTC using two values for ρ (the maximum angular gap required): $\rho = \frac{2}{3}\pi$ and $\rho = \frac{\pi}{2}$.

– *Homogeneous*: we have also considered the situation in which no topology control is used. In this case, the value of the transmitting range is defined as the 0.99 quantile of the empirical distribution of the critical transmitting range (see Section 6.1).

First, we have evaluated the energy cost of the different protocols. For the k-NEIGH and CBTC protocols, we have considered both the result of the Phase 1 only (without pruning), and of the protocols with the pruning stage implemented. The rationale for this investigation is that in some situations (e.g., high mobility scenario), implementing the pruning step could be very difficult. We have considered two values for the distance-power gradient α, i.e., $\alpha = 2$ and $\alpha = 4$. The value of the distance-power gradient has a strong influence on the pruning phase of k-NEIGH and CBTC, which are essentially based on triangular inequalities on the power function: the higher α, the more edges are pruned.

In Figure 6, we show the energy cost (normalized with respect to the cost of the MST) of the different protocols when $\alpha = 2$, for increasing values of n. As can be seen, the Phase 1 of our k-NEIGH protocol performs much better than that of CBTC, for both values of ρ: for $n = 1000$, the energy cost of k-NEIGH is 83% lower than Phase 1 of CBTC with $\rho = \frac{2}{3}\pi$. Compared to the case of no topology control, k-NEIGH-Phase 1 provides an improvement of 16% when $n = 10$, and of 77% when $n = 1000$. Observe that CBTC performs worse than the case of no topology control: this is due to the fact that the maximum transmitting range R_n used in CBTC is 50% larger than the 0.99 quantile of the critical transmitting range distribution used in the Homogeneous protocol. In case a lower value of R_n would be used, CBTC would perform better, at the expense of an increased probability of gen-

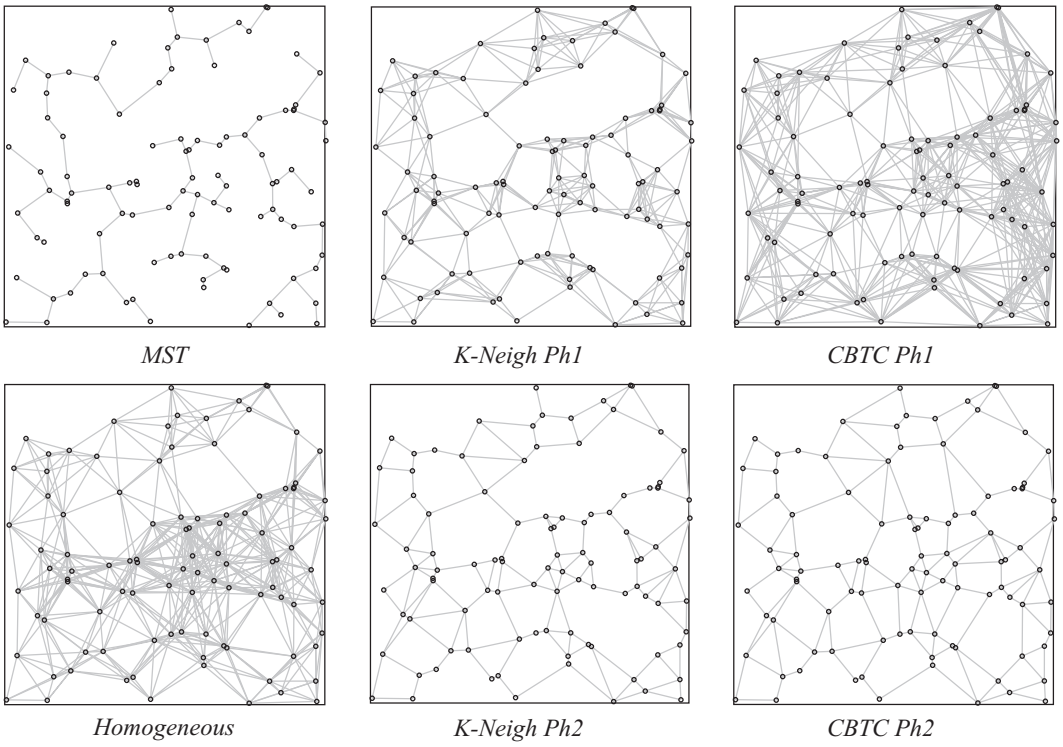

Figure 7: Sample topologies produced by the different topology control protocols with $n=100$. In CBTC, ρ is set to $\frac{2}{3}\pi$.

erating a disconnected graph. Contrary to CBTC, our k-NEIGH protocol is almost independent of the choice of R_n: using the same value of R_n of CBTC, k-NEIGH is several times better than Homogeneous. This is due to the fact that in CBTC, several nodes (those lying on the boundary of the region) are expected to transmit at maximum power, since it is very unlikely that the required angular gap ρ can actually be achieved. Conversely, in k-NEIGH we require the connection to the k nearest nodes, independently of their direction.

The implementation of the pruning stage decreases the energy cost significantly in both k-NEIGH and CBTC protocols (see Figure 6 – right). Nevertheless, k-NEIGH still performs better than CBTC: except for small network sizes ($n=10$ to 20), the energy cost of k-NEIGH is about 20% smaller than that of CBTC. The experiments show that the topologies generated by k-NEIGH can be as much as 87% more energy efficient than in those with no topology control, while they are at most a factor of 2.28 away from the cost of the "optimal" MST topology. A sample of the topologies generated by the various protocols for $n=100$ is shown in Figure 7.

In Figure 8 we report the average logical (left) and physical (right) node degree of the topologies generated using k-NEIGH and CBTC. As in the previous case, we have considered both protocols without and with the pruning stage implemented. From Figure 8, it is evident that k-NEIGH-Phase 1 outperforms CBTC-Phase 1 in terms of both logical and physical degree. Observe that in k-NEIGH we have the upper bound k on the number of *physical* neighbors of any node, which holds for Phase 1 also. On the contrary, the result of [27] on the maximum number of neighbors (which, we recall, is 6) regards the topology generated by CBTC *after* pruning; furthermore, the upper bound is on the number of *logical* neighbors. Finally, note that k-NEIGH performs better than CBTC also when Phase 2 is implemented.

We have performed the same simulations with $\alpha = 4$. The results of these experiments, which are not reported for lack of space, confirmed on a larger scale that k-NEIGH performs better than CBTC in terms of energy cost, logical and physical average node degree. In terms of energy cost, k-NEIGH-Phase 1 performs as much as 97% better than CBTC-Phase 1, and as much as 94% better than the case of no topology control. With Phase 2 implemented, k-NEIGH is as much as 29% better

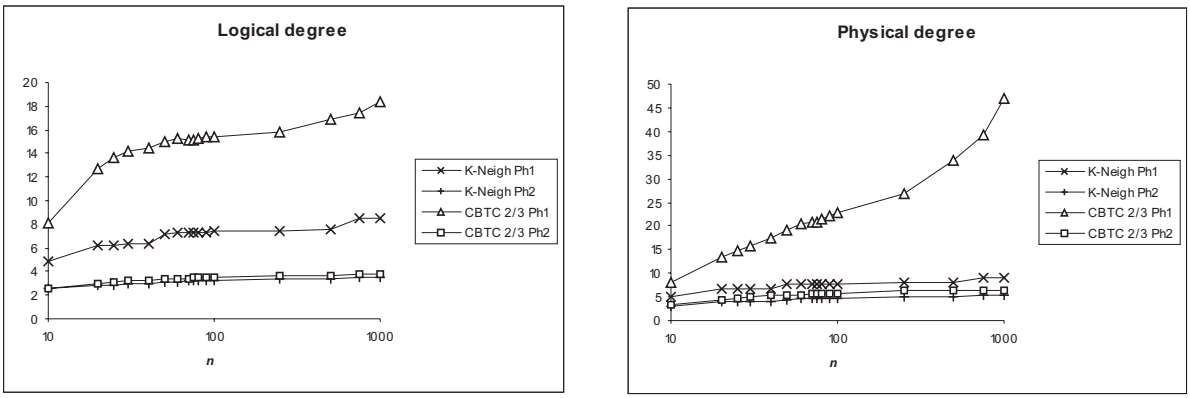

Figure 8: Average logical (left) and physical (right) degree of the topologies generated by the k-NEIGH and CBTC protocols. Values on the x-axis are reported in logarithmic scale.

than CBTC, and as much as 98% better than the case of no topology control.

Overall, the results of this last set of experiments have shown that:

– k-NEIGH-Phase 1 performs significantly better than CBTC-Phase 1. Essentially, this is due to the fact that, contrary to the case of CBTC, after the execution of k-NEIGH-Phase 1 relatively few nodes are expected to transmit at maximum power. To some extent, this seems to indicate that k-NEIGH is well suited to be implemented in a high mobility scenario (see Section 7 for further discussion o this point).

– If pruning is implemented, k-NEIGH still performs better than CBTC, in terms of energy cost, as well as logical and physical average node degree.

7 Future work

An important topic for future work is to adapt the k-NEIGH protocol to deal with mobility. In a mobile network, the topology is continuously changing and the topology control protocol must be reexecuted periodically. A quantitative evaluation of k-NEIGH in mobile environments is beyond the scope of this paper. However, here we present a brief qualitative discussion of how the protocol can be adapted for mobile environments and how it compares to other algorithms in this case.

In the k-NEIGH protocol presented herein, the number of neighbors is set to a very precise value. If this protocol is extended to mobile networks, it would be quite expensive to control the neighbor set size so precisely: this could require reexecution of the protocol each time the neighbor set changed. Instead, we adopt the approach taken in MobileGrid and LINT, where low and high water marks are specified such that the neighbor set size falling below the low water mark or exceeding the high water mark causes the protocol to be reexecuted. Since the value of k determined in Theorem 2 is sufficient for connectivity, it is a likely candidate for the low water mark for the mobile version of the protocol. The high water mark could be determined based on the velocity of nodes and the expected transmitting range to ensure that the protocol does not need to be reexecuted too often. The initial value of the desired number of neighbors in the protocol should then be set to the average of the low and high water marks.

Since any topology control protocol needs to be executed periodically in a mobile network, the energy consumed during the protocol execution becomes even more important than the "quality" of the topology produced. Thus, we believe that the benefits of the k-NEIGH protocol will be even greater in this situation. This is because ours is the only known protocol with a proven upper bound on the number of messages exchanged during its execution. The number of messages exchanged in the k-NEIGH protocol is expected to be far lower than CBTC in practice since CBTC iteratively sends messages in a first phase and then sends even more messages during a second optimization phase. Our future work will focus on specification and evaluation of a mobile k-NEIGH protocol, with the goal of showing that the signifcant benefits shown herein for the stationary version of the protocol are maintained in mobile environments.

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